Interface mobilities – theory and applications

Björn A. T. Petersson
Institut für Technische Akustik, Technische Universität Berlin

Introduction

The analysis of vibro-acoustic transmission in built-up structures often relies on the assumption of point-like connections between the substructures. Although the general matrix formulation is comparatively well developed, the analyst is referred to numerical or experimental techniques to determine the dynamic characteristics of the constituents, except for systems with elementary geometries. Already for a limited number of connection points, however, the computations or measurements and the associated interpretation can become cumbersome and simplifications are both necessary and desirable. Complication also arises when two or more subsystems are joined at line- or surface-like interfaces, which are large compared with the governing wavelength. Obviously, a discretisation of the interface is an intuitive possibility but this implicitly leads to a substantial set of 'contact points' with the same drawbacks as those of the truly discrete case.

In a suite of recent publications an approach to treat large, continuous, closed contour interfaces has been addressed from various viewpoints, e.g. [1-5]. It is demonstrated that with the introduction of the concept of interface mobility [4], a concept intimately related to the class of Fredholm integral equations, a significant simplification can be obtained for small and intermediate Helmholtz numbers. The main reason being that the transmission problem is transposed into that of an equivalent, single point and single component of motion and excitation. In this light and guided by the ortho-normal basis for the interface mobility, its applicability to cases with multiple point connections between the subsystems becomes interesting. Of foremost interest is the application of interface mobilities in conjunction with source characterisation as recognised in [6]. Thence, most of the inherent complications regarding multi-point or surface installed source systems are removed.

Some fundamentals

For continuous velocity fields and force distributions over a closed contour, the interface mobility is defined as,

$$Y_{pq} = \frac{1}{C^2}\oint_C\oint_C Y(s\,|\,s_0)e^{-ik_p s}e^{-ik_q s_0}\,ds\,ds_0 \qquad (1)$$

Consider a multi-point interface between a source and a receiver structure such that a closed contour can be formed, passing all the points. Upon theoretically smearing the interface forces, a continuous force distribution is established, $F(s) = \sum F_m \delta(s - s_m)$, such that the coefficients of the associated Fourier series become, $F_q = \left(\sum F_m e^{-ik_q s_m}\right)/C$, where C is the perimeter of the contour. Similarly, the vibrations along this contour can be expanded as, $v_p = (1/C)\int_C v(s)e^{-ik_p s}ds$. The general expression for the complex power transmitted from the source to the receiver is then found to be given by, $Q = C^2\left(\sum Y_{pq}F_p^* F_q\right)/2$. With the cross-order-terms small compared with the paired terms, the transmission problem and the source characterisation is simply subdivided into a sequence of orders, for instance,

$$Q = \frac{1}{2}(Y_{00}|F_0|^2 + Y_{11}|F_1|^2 \cdots + Y_{-1-1}|F_{-1}|^2 \cdots)(2)$$

From previous studies it is seen that for small Helmholtz numbers, kR, where R is a typical radius or dimension of the interface, the cross-order terms are generally small compared with those of zero and first order. For large Helmholtz numbers, moreover, the cross-order terms asymptotically vanishes. This means that for many engineering applications, the formal simplicity of the single-point, single-component case can be retained and terms included as required. Finally, it opens a viable scheme for source characterisation since all paired orders can be treated as individual source contributions.

Experimental comparison

In order to gain some experience of working with interface mobilities an experiment was undertaken with an ordinary ventilation fan having four mounting points, installed on a concrete slab. Two installation positions were included, one at the centre, the other close to an edge of the plate and all the ordinary point and transfer mobilities of both the fan and plate were measured. For brevity, only the force and velocity components perpendicular to the plate were considered. In addition were measured the velocities at the mounting points of the operating fan when dynamically free. From this data set, the active power transmitted to the plate was determined, first, by means of the general matrix formulation and second, by using the interface mobilities. The comparison for the centre position is shown in Figure 1 where the active, transmitted power is displayed versus Helmholtz number. For this installation, the zero order interface mobility manages to capture the power transmission up to a Helmholtz number of about eight. Above this point higher orders are required to bring down the overestimation and thereby more mounting points then the diagonally located two used in this examination, have to be included.

Concluding remarks

The application of interface mobilities to the four mounting point fan installation demonstrates their capability to assess the structure-borne sound transmission also for discrete connections between sources and receivers. Since the measurement effort can be limited for low orders of interface mobilities, the risk that random measurement or numerical processing errors corrupt the estimation is markedly reduced. Finally, the approximate version of the interface mobility is directly suited for source characterisation [7].

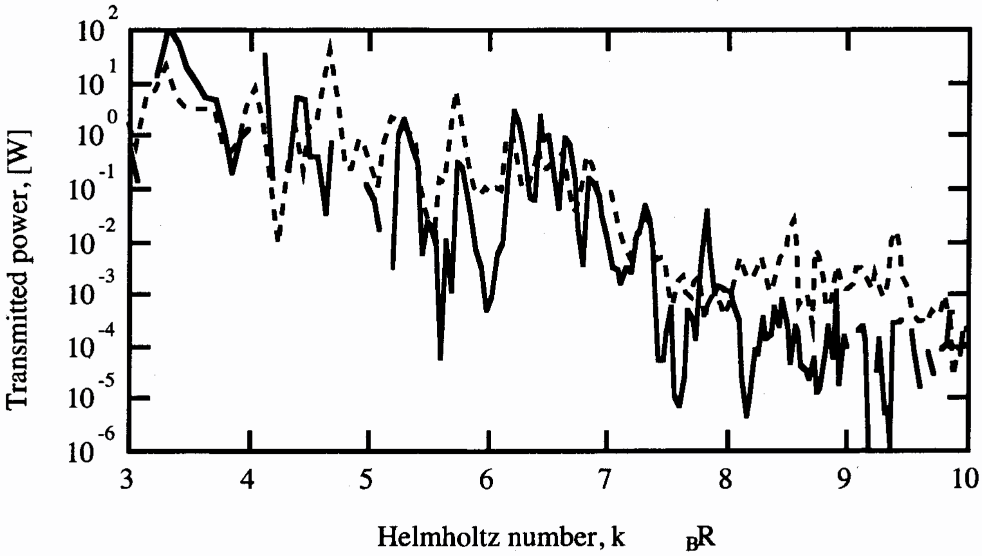

Figure 1. Transmitted power, (———) matrix and (- - - -) zero order interface mobility calculations.

References

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