DEVELOPMENT OF ACCELEROMETERS WITH BENDING ELEMENTS

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High-sensitive sensors of elastic waves are needed for development of modern methods of acoustic nondestructive testing and seismoacoustic prospecting. Piezo-accelerometers are the most widely used for measurements of elastic wave parameters. In this paper the basic requirements and principles of designing of high-sensitive piezo-accelerometers for acoustic diagnostics having bending bimorph elements and central cylindrical support are considered. Such a sensors have many advantages by their parameters over accelerometers using other forms of piezo-element deformation (compression-tension, shift), and this is discussed in the paper.

ACCELEROMETER DESIGNING

Piezo-accelerometers with oscillations of bending are characterized by high sensitivity and rather small dimensions and weight, by minimum sensitivity to strains of the fastening place and to cross oscillations, and by simplicity of design. From the other hand, beam and membrane constructions are insufficiently strong and have rather low eigenfrequencies. Just for this reason transducer designers perform search of successful construction schemes with oscillations of bending allowing with other conditions being equal to increase mechanical strength and impact resistance of gauges. One of such schemes basing on bimorph sensing element (SE) of the “mushroom” shape without an inertial element (see Figure 1) is used already more than for 20 years (high-sensitive accelerometers, resonant-type gauges of detonation for automobile engines, recorders, etc. [1]). This scheme can be used in different types of measurements, and consequently looks as the most preferable in comparison with analogs containing inertial elements.

![Fig.1. Principle construction scheme of a piezo-accelerometer with oscillations of bending: 1 - piezoelement; 2 - elastic membrane; 3 - basic core; 4 - housing; 5 - electrical outputs.](image)

Absence of additional inertial element except the piezoelement itself 1, and the elastic membrane 2, design simplicity and configuration rationality provides required strength and wide dynamic (up to 120 dB) and frequency (up to several tens of kHz) ranges.

In this paper the basic principles of calculation and designing of piezoelectric sensors with oscillations of bending are considered by the example of a piezo-accelerometer with bimorph SE of the “mushroom” shape.

Values of effective mechanical stresses, forces or moments at corresponding contour of plate should be specified, when calculating any element as a thin plate subjected to bending strains. At their small deflections, the three-dimensional problem can be reduced to the two-dimensional one. Taking into account the assumptions for thin plate, the values of effective mechanical stresses, forces or moments at corresponding contour of plate should be specified, when calculating any element as a thin plate subjected to bending strains. At their small deflections, the three-dimensional problem can be reduced to the two-dimensional one. Taking into account the assumptions for thin plate, the equations for mechanical stresses and electric induction of piezo-ceramic material become as follows [2,3]:

\[
\sigma_{rr} = \frac{s_{11}}{s_{11} + s_{12}} \left( \frac{\partial^2 u_r}{\partial r^2} - \frac{1}{r} \frac{\partial u_r}{\partial r} \right),
\]

\[
\sigma_{\theta\theta} = \frac{s_{12}}{s_{11} + s_{12}} \left( \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{\partial^2 u_r}{\partial r^2} \right),
\]

\[
D_z = \frac{d_{11}}{s_{11} + s_{12}} \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} \right) + \left( \epsilon_{11} - \frac{2}{s_{11} + s_{12}} \right) E_z.
\]

The mechanical boundary equations can be obtained in supposition of zero energy flux through corresponding contour of plate, when the contour either is fastened or is free. In the latter case, (\( \sigma_{rr} = 0 \)) the forces of electromechanical nature work. Electrical energy imparted to piezo-element (PE) is transformed into mechanical energy. At fastened contour the PE is electromechanically passive.

At formulation of boundary conditions the multilayer SE should be changed for an equivalent plate having thickness \( h \) and density \( \rho \), Poisson ratio \( \mu \), and cylindrical rigidity \( C \).

On the outside contour \( R_o \) of the plate the bending moment in radial direction \( M_r \) and cross force \( Q_z \) are equal to zero, and on the interior contour \( r_o \) it should be assumed that deflection \( u_r \) is equal to zero, and the turn angle is \( \psi = \frac{\partial u}{\partial r} \).

The problem of calculation of multilayer SE is reduced to determination of location of a neutral surface and a value of equivalent bending rigidity. The effective mechanical stresses in sections of \( n \)-th layer of a plate can be calculated as well as for one-layer plate.

As an example we present the system of the boundary mechanical equations for SE as a round three-layer plate rigidly fastened on the central cylindrical support and consisting of piezoceramics layers, glue and metal (see Fig. 4). Density \( \rho \) and thickness \( h \) of the effective plate:
\[ \rho = \frac{\rho_p \cdot h_p + \rho_m \cdot h_m + \rho_e \cdot h_k}{h} \]

\[ h = h_p + h_k + h_m \]

where \( \rho_p \) is the piezoceramic density; \( h_p \) - thickness of piezoceramic disk; \( \rho_m \) - density of material of elastic element; \( h_m \) - thickness of elastic element; \( \rho_e \) - glue density; \( h_k \) - thickness of glue layer.

Equivalent rigidity of multilayer SE, as well as equivalent elasticity modulus \( E \) and Poisson ratio \( \mu \) are determined from the condition that the sum of bending moments in the side section of the multilayer plate is equal to the bending moment of equivalent one-layer plate \( M_f \) [2,3]:

\[ D = \frac{s_{11}}{s_{11} - s_{12}} \int s_{12} y(y + \zeta) dy + E_k \frac{1}{1 - \mu_k} \int s_{12} y^2 (y + \zeta) dy + E_m \frac{1}{1 - \mu_m} \int s_{12} y^3 dy \]

\[ M_f = \frac{d \phi}{dr} \frac{\rho \cdot \varphi}{r} \left( \frac{s_{11}}{s_{11} - s_{12}} \int s_{12} y(y + \zeta) dy + E_k \frac{1}{1 - \mu_k} \int s_{12} y^2 (y + \zeta) dy + E_m \frac{1}{1 - \mu_m} \int s_{12} y^3 dy \right) \]

where \( \varphi \) - angle of turning of the neutral surface of the plate; \( z \) - distance from the neutral surface to the considering plane of plate; \( \zeta \) - distance from the neutral surface to the axis beginning.

It should be noted that the neutral surface of multilayer plate is general to the one-layer plate, does not generally coincide with the median surface and, hence, the neutral axes of sections do not pass through geometrical centers of plate. Distance \( \zeta \) from the neutral surface to the axis of measurement beginning (located in the plane where piezoelement is glued to elastic element) can be found from the condition of absence of normal stresses and forces at the neutral surface \( \sigma_{z} = 0 \), \( \sigma_{zz} = 0 \) [2,3]:

\[ \zeta = \frac{E_m \cdot h_m}{1 - \mu_m} - \frac{s_{11}}{s_{11} - s_{12}} \cdot h_p^2 + \frac{E_k}{1 - \mu_k} \cdot h_k^2 + 2 \cdot \frac{E_k}{1 - \mu_k} \cdot h_k \cdot h_m \]

Obtained dependencies of SE displacements and mechanical stresses of PE are the basis for the calculation of transducer characteristics. It should be noted that the neutral surface of multilayer plate, as against to the one-layer plate, does not generally coincide with the median surface and, hence, the neutral axes of sections do not pass through geometrical centers of plate. Stresses in sections of the \( n \)-th layer of the bending multilayer plate can be calculated like for a one-layer plate having the same rigidity as the considered multilayer plate.

In the electroelasticity problems the boundary conditions can be divided into two groups of conditions of conjugate fields on the piezoceramic surface: mechanical and electrical. The boundary conditions for the mechanical component are reduced to usual relations of the elasticity theory. Thus, if the vector of mechanical stresses is specified on the surface of a body, it is necessary to take advantage of dependencies of the generalized Hooke’s law and to equate stresses on the surface of body to the specified values. The obtained three equations relate components of displacement vector and electrostatic potential. The electrical boundary condition is formulated for the electrical component of conjugate field, and it depends on method to pick electrical energy off PE. It should be noted that generally in PE a non-homogeneous mechanical field is formed, and only in the limits of elementary layers (volumes) the mechanical stresses are homogeneous. To exclude ambiguity of the mechanical conditions, it is necessary to make equations for forces effecting on PE edges or for mechanical energy entering PE through corresponding edge.

Generally the input circuits of amplifying-transforming equipment include a cable line having capacity \( C_e \) and inductance \( L_e \), as well as input electrical resistance \( R_e \) and capacity \( C_e \) connected parallely with the cable. Determination of electrical voltage at input of the electronic unit is reduced to solution of the system of equations.

\[ L \cdot \dot{I} + r \cdot I + \frac{1}{C_e} \int I dt + R_e \cdot I_n - U = 0 \quad R_e \cdot I_n + \frac{1}{C_e} \int (I - I_n) dt = 0 \]

Solution of these equations gives the amplitude-frequency characteristics of the system. As an example in Fig.4 it is shown calculated AFC of piezoelectric sensor unit of the half-resonant type for the mode of electrical no-load operation (a) and the mode of loading of input circuits of amplifying-transforming equipment (b).

The presented technique for calculation of piezoelectric sensors with sensitive element subjected to bending deformations allows one to optimize the design-assembly parameters of sensors. Results of calculations are in good agreement with experimental data. This work was supported in part by the International Science and Technology Center (project 1369). Authors appreciate DFG for support.

REFERENCES