Introduction

This is a tutorial contribution concerning the use of the classical Sonar equation in a stochastic multipath environment. The simple form of the Sonar equation, which is widely used to estimate the detection of objects submerged in water or the ocean sediment reads:

\[ EL = SL - 2 * TL + TS, \]

\[ \text{if } \ EL > \max(RML, NML) \]

with \( EL \) Echo Level, \( SL \) Source Level, \( TL \) Transmission Loss, \( TS \) Target Strength, \( RML \) Reverberation Masking Level, \( NML \) Noise Masking Level (capital letters denote variables in dB).

The only quantity which describes the object itself is the target strength, and all signal processing gains of modern highly sophisticated sonars can be incorporated into the masking levels. However, the target strength is defined independent of the environmental conditions encountered during the actual detection. The aim of this tutorial is to illustrate, that the received echo-structure and echo-strength may be influenced considerably by the environmental multipath conditions (e.g.,\([1]\)).

Target strength

The term target strength \( TS \) of an object is defined for an CW signal of infinite duration with frequency \( f \) as the ratio of incident intensity \( I_i \) to outgoing intensity \( I_o \):

\[ TS = 10 * \log \left[ I_o(\alpha_2, \beta_2, f) / I_i(\alpha_1, \beta_1, f) \right], \]

with \( \alpha \) denoting the azimuth angle in the horizontal plane and \( \beta \) the elevation angle from this plane. The intensity is measured in the farfield at range \( r \) with \( L^2/\lambda \ll r \), where \( L \) denotes the length of the object and \( \lambda \) the acoustic wavelength. This relates to the free field scattering amplitude or reflection coefficient \( \mathcal{R} \) of the object by \( TS = 10 * \log(\mathcal{R}^2) \) with \( \mathcal{R}(\alpha_1, \beta_1, \alpha_2, \beta_2, f) \).

In the real ocean the sound is transmitted over larger distances to the target via a multitude of transmission paths simultaneously. Hence several acoustic waves arrive at the target with different amplitudes, phases and travel-times. The coherent sum of all contributions from the multipaths generate the effective target strength for this detection.

Two approaches are conceivable to compute the effective target strength. Firstly, in the direct method, the sound field is determined to every point on the target and the appropriate reflection and scattering is considered. This requires to incorporate the object into the sound propagation program. Secondly, in the indirect method, the bistatic target strength of the object is computed separately and used within a sound propagation code. The indirect method will be used here.

The sound propagation is assumed to be azimuthally invariant, i.e. independent of the horizontal direction.

The sound field at location \((x, z)\) is given by \( p(x, z, f) \). This quantity is normalized by the source strength so that \( p \) denotes the transmission loss. To compute the propagation from object/target to the receiver the principle of reciprocity is used:

\[ p_1([x_1, y_1]) \rightarrow (x_2, y_2) = p_2([x_2, y_2]) \rightarrow (x_1, y_1), \]

where the angular source and receiver patterns are to be exchanged accordingly. This implies stationarity, which may not only be assumed for time invariant oceanic processes but for time variant conditions as well, if the range is sufficiently large and the proper expectation values can be defined.

Using reciprocity implies co-located source and receiver, hence the term bistatic refers to different vertical angles. The effective (vertically) bistatic echo strength \( \phi \) results from integrating over all possible paths from source to target and target to receiver:

\[ \phi = \int p_1(\alpha_1, \beta_1) \mathcal{R}(\alpha_1, \beta_1, \alpha_2, \beta_2) \int p_2(\alpha_2, \beta_2) d\beta_1 d\beta_2 \]

For monostatic source receiver geometry the expression simplifies through \( R_1 = R = R_2 \) and \( \alpha_1 = \alpha_2 = \alpha \), but still is a vertically bistatic problem.

Soundfield at the target

For the target strength computation the reference location of the field incident on the target lies typically in the center of the target object, which serves as well as the reference location for the transmission loss values. In this approximation the sound field at range \( x \) will be computed for a depth interval \( dx \) which should not be smaller than the vertical dimension of the object. The requirement, that the variation of the average sound field is small over the dimension of the object is met for most realistic conditions.

![Figure 1: Soundspeed profile and bottom contour](image)

In the following examples sound is propagated through a typical shallow water sound scenario (fig. 1) with source depth 70 m and frequency 3 kHz [2]. The angular distribution of the ray arrivals is displayed for spatially constant and spatially stochastic speed conditions in figs. 2 and 3 respectively. The transmission loss amplitude is colour coded.
In the constant sound speed case the late arrivals correspond to steep rays contacting both boundaries and pairs of arrivals can clearly be distinguished. For a stochastic sound speed environment with large fluctuations this order is grossly disturbed and the loss increased, because the larger angles suffer larger reflection loss at the bottom.

Figure 2: Ray arrivals versus retarded time and angle for constant conditions, range 20 km, depth 90-100 m

Figure 3: Ray arrivals versus retarded time and angle for stochastic conditions, range 20 km, depth 90-100 m

The target object considered here is a pair of spheres, with radius of 0.5 m with a vertical separation of 1.5 m between the spheres. The bi-static reflection coefficient is computed at 3 kHz for hard boundary conditions in the plane wave approximation, and displayed with maximum normalised to one in fig. 4, where the two angular directions are elevation angles in the xz-plane relative to the x-axis. The pattern is caused by the interference of contributions reflected from the two spheres.

Figure 4: Bi-static reflection coefficient (maximum normalized to one) in PWA approximation of the spheres

To compare the influence of the environment on the effective bi-static reflexion the argument of the integral is displayed in fig.5 for case of constant sound speed conditions (with \(\alpha_1 = \alpha_2 = \text{const. and normalized to one} \).

Figure 5: Scaled argument of integral \(\log([p_1 \ Re \ p_2])\) for propagation in fig.2

In the same normalization the argument of the integral for the effective bi-static reflexion coefficient is displayed in fig. 6 for the stochastic environment.

Figure 6: Scaled argument of integral \(\log([p_1 \ Re \ p_2])\) for propagation in fig.3

For both scenarios the received energy is dominated by the multipath structure, i.e. by the off-diagonal terms in the integral. In this special case the effective target strength is \(TS_{det} = -1 \) dB and \(TS_{stc} = -4 \) dB for the deterministic and stochastic case and the difference to the nominal strength of \(TS_0 = 0 \) dB is small. However the signal structure and ambiguity function are distinctly different.

Conclusion

Depending on multipaths generated by the environment the sound samples a different section of the bi-static reflection matrix \(R(\beta_1, \beta_2)\) with different amplitudes. And hence the intensity to be used in the TS definition for practical purposes implies the integral over all contributions due to the multipath structure.
