In this paper the analysis of the sensitivity of different frequency domain photoacoustic approaches on the changes of the thermal diffusivity of the measured material is shown and discussed. For the analysis of accuracies and sensitivities the least square fitting method was applied. From the point of view of determination of the thermal diffusivity from the frequency domain amplitude and phase characteristics the range of frequencies applied is one of the most important factors that determines the accuracy of its determination. The optimal range of the frequencies applied is determined by the thermal diffusivity of a measured material and its thickness and is determined by the characteristic frequency \( f_0 = \alpha/(2\pi d^2) \).

The sensitivity of a given approach was estimated from the minimum of the Sum of Square Errors defined below:

\[
SSE = \frac{1}{N-1} \sum_{i=1}^{N} (PAS(f_0, f0 \cdot (1 + p)) - PAS(f_0))^2
\]

where \( p \) is the percentage change of \( f_0 \) and it is represented by solid lines on the figures. SSE values were compared with the values of the SSEN parameter defined below (dotted line on the figures):

\[
SSEN = \frac{1}{N-1} \sum_{i=1}^{N} (PAS(f_0, f0 \cdot (1 + p)) - PASN(f_0))^2
\]

SSEN is the sum of square errors between the theoretical value calculated for \( f_0 \) with the Noise. For amplitude characteristics the normal distribution type Noise in the range \( \pm 2\% \) of the amplitude at the frequency \( f= f_0 \) was taken. For phase characteristics the normal distribution type Noise in the range \( \pm 2\text{deg} \) was taken.

The minimum value of SSEN gives the average, expected value of \( f_0 \). The value of SSE equal to the value of SSEN in the point of minimum gives the accuracy of determination of the \( f_0 \) \((f_0 \in [p_{\text{min}}, p_{\text{max}}])\).

**Investigated cases**

1. **Front amplitude method**: \( f \) range \( 0.1f_0 \text{-}10f_0 \).

   ![Amplitude of PA signal vs. the frequency of modulation](image1)

2. **Amplitude rear method**: \( f \) range \( f_0 \text{-}10f_0 \).

   ![SSE and SSEN vs p](image2)

3. **Amplitude Ratio \( S_f/S_r \) method**: \( f \) range \( f_0 \text{-}10f_0 \).

   \( S_f \) is the amplitude of the PA signal measured in the front configuration regime. \( S_r \) is the amplitude of the PA signal measured in the rear configuration regime.

   ![Amplitude of PA signal vs. the frequency of modulation](image3)
Fig. 3 a) Amplitude ratio of $S_f$ and $S_r$ PA signals vs. the frequency of modulation b) SSE and SSEN vs $p$ - the percentage change of $f_0$

4. Phase front method: $f$ range 0.1$f_0$–10$f_0$

Fig. 4 a) Phase of PA signal vs. the frequency of modulation b) SSE and SSEN vs $p$ - the percentage change of $f_0$

5. Phase rear method: $f$ range $f_0$–10$f_0$

Fig. 5 a) Phase of PA signal vs. the frequency of modulation b) SSE and SSEN vs $p$ - the percentage change of $f_0$

6. Phase Lag method: $f$ range $f_0$–10$f_0$

Fig. 6 a) Phase Lag of PA signal vs. the frequency of modulation b) SSE and SSEN vs $p$ - the percentage change of $f_0$

Summary

The table below shows the comparison of percentage errors of determination of thermal diffusivities obtained by different photoacoustic measurement methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>$p_{\text{min}}$</th>
<th>$p_0$</th>
<th>$p_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Rear</td>
<td>-7%</td>
<td>0%</td>
<td>+7%</td>
</tr>
<tr>
<td>Phase Lag</td>
<td>-8%</td>
<td>0%</td>
<td>+9%</td>
</tr>
<tr>
<td>Phase Front</td>
<td>-28%</td>
<td>0%</td>
<td>+20%</td>
</tr>
<tr>
<td>Amplitude Front</td>
<td>-15%</td>
<td>+10%</td>
<td>+30%</td>
</tr>
<tr>
<td>Amplitude Rear</td>
<td>-60%</td>
<td>-35%</td>
<td>Not defined</td>
</tr>
<tr>
<td>Amplitude Ratio $S_f/S_r$</td>
<td>-45%</td>
<td>-20%</td>
<td>Not defined</td>
</tr>
</tbody>
</table>

It comes out from the comparison that the most accurate results can be obtained by the phase methods ($p_0=0\%$). Amplitude methods give systematic errors in the range from +10\% to −35\%.

Literature