Transmission loss of inhomogeneous plates with local resonators: Methods of theoretical modelling

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It remains a challenge to design plates with a high transmission-loss in the mass-law frequency range without increasing the total mass. Recently, a remarkable concept has been studied by a group of physicists at the University of Hong Kong [1]. A plate made from a 'locally resonant sonic material' consisting of lead spheres, which are coated with silicone rubber and embedded in epoxy, showed an astounding performance (see Fig. 4). On the way to a quantitative understanding two methods of theoretical modelling are studied below.

Modelling with periodically structured thin plates

In the first model the epoxy plate with the randomly dispersed lead spheres is idealised as a thin plate with inhomogeneities periodically arranged in a hexagonal manner. This kind of structure may be handled by the computer code HYPERAKUS, which is based on a generalisation of Cremer's treatment of homogeneous thin plates [2]. Since there are no circles available in the current version, a square geometry has been used instead.

The periodic structure causes diffracted waves in addition to the reflected and transmitted waves with homogeneous plates. These diffracted waves are propagative above their cut-on frequencies, whereas below they decay exponentially with distance from the plate surfaces ("nearfields"). For the considered structure the first cut-on frequency is beyond 10 kHz, therefore the spike in the resulting transmission loss curve (not shown here) is exclusively due to nearfields. A behaviour similar to the measured result is found: an improvement over the mass law followed by a deterioration. Unfortunately, this result is not fully converged due to limited computing capacity. Therefore, the structure has been simplified further to one-dimensional periodicity (Fig. 1).

Now a rather good convergence could be achieved with the maximum possible number of 871 nearfields. The shape of the transmission loss curve is the same as in the previous case with two-dimensional periodicity (Fig. 3). A visualisation of the plate motion at selected frequencies reveals: At 328 Hz the lead and epoxy parts move in phase albeit with different amplitudes. At 800 Hz, however, they move against each other: a plausible explanation of the transmission-loss maximum. Further increase of the frequency reduces the amplitude of the lead parts, which eventually seem to come to rest. This is the reason why the transmission loss curve remains below the straight line of the mass law.

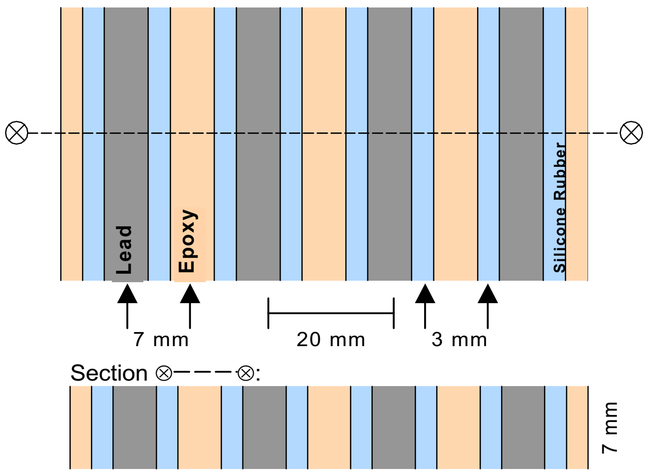

Fig. 1: Structure for 1D HYPERAKUS calculation

Although a more realistic modelling of the measurement of Liu et al. [1] is not possible with HYPERAKUS, the essential physics at the transmission-loss maximum is believed to be the same. Closer quantitative agreement may be expected with a model describing thick plates with periodic structure [3], where embedded spheres can be included without principal problems. Unfortunately, the required computing capacity is still much higher than for the two-dimensional HYPERAKUS case.

Modelling with harmonic oscillators

The simplicity of the described displacement patterns provokes the search after a comparatively simple theoretical model without the necessity of extensive number crunching. Fig. 2 shows such a model consisting of coupled harmonic oscillators (cf. also [4]).

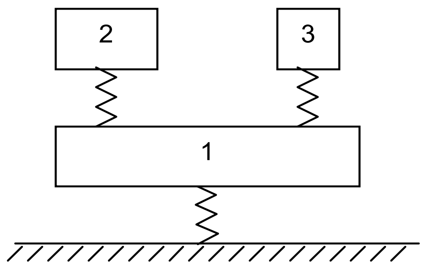

Fig. 2: Harmonic oscillator model with "Master" (1) and "Slaves" (2 and 3)

The first harmonic oscillator ("master HO") corresponds to the epoxy part of the plate and the "slaves HO" represent the lead spheres, which are elastically coupled to the "master" by rubber springs. The excitation amplitude of each HO is proportional to its exposed area. This area is zero in the case of the embedded spheres, i. e. the spheres are only excited indirectly by the "master" and not by the incident acoustic wave. In the case of the HYPERAKUS models the lead parts are directly excited, too. For normal sound incidence and identical coated lead spheres one "slave" is sufficient.

The agreement between such a HO calculation and the HYPERAKUS result is almost perfect (Fig. 3). A significant difference exists only around the transmission-loss minimum. This probably indicates that the damping description with the damping force proportional to the velocity of the HO mass should be modified.

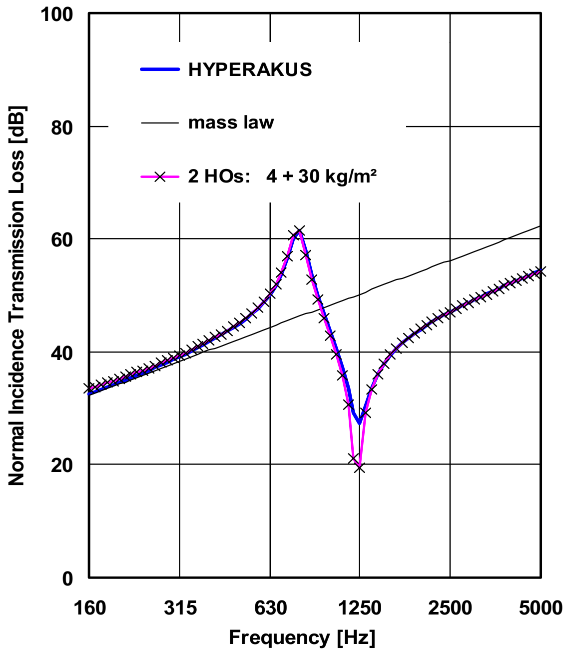

Fig. 3: Results from HYPERAKUS and from harmonic oscillator model

Can we also reproduce the measurement? From Fig. 4 it is obvious that there is no quantitative agreement. However, the trend is roughly correct. An improvement can be achieved by adding an imaginary mass to the lead HO, but before a more detailed discussion of the merit of this *ad-hoc* action it should be given a reliable theoretical basis.

Conclusions

Remarkable improvements of the transmission loss of plates in the mass-law frequency range can be accomplished by local resonators with a size much smaller than the wavelengths involved. The theoretical

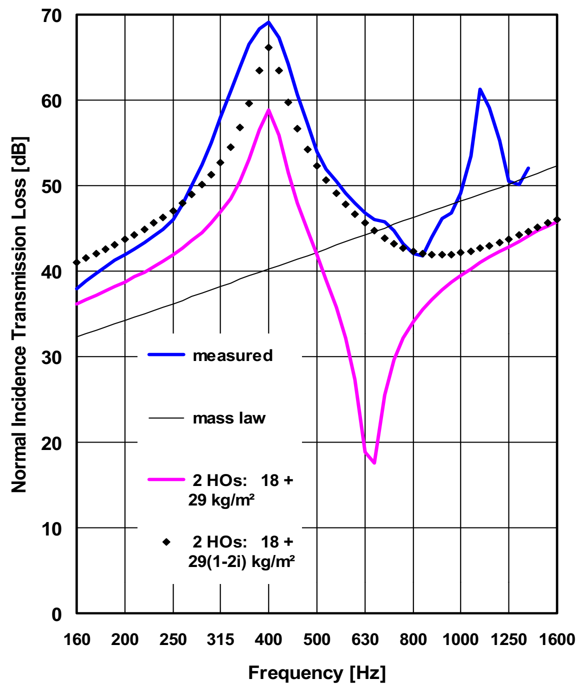

Fig. 4: Measurement [1] and results from harmonic oscillator model

description of this effect may proceed along two lines, either by a continuum-mechanical approach, which requires considerable numerical resources, or by comparatively simple models of coupled harmonic oscillators, which admittedly need refinement for a better agreement with measured results. Further steps should include more realistic damping terms and consideration of three-dimensional effects.

References

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