Calculation of the reflected field from an impedance plane

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Introduction

Many room acoustic models have been developed to describe the various aspects of room acoustic filed. As far as the physics of the process is concerned, interference for instance should be modelled by complex reflections ("phase image model"). At each reflection of sound wave from wall surface, a complex reflection is used to account for the amplitude and phase change. It is shown [1] that this type model gives much better representation of the energy impulse response in a given frequency band than the conventional model with simple energy addition. An important issue to be addressed in modelling complex reflections from a room surface is, of course the correct choice of reflection coefficient. The reflection of radiated sound is frequently described by the plane wave reflection coefficient, despite the fact that the source is of spherical symmetry. It leads to a good agreement with experimental results except for grazing incidence sound or surfaces of high absorption coefficient. Such situation can be realised in small rooms at low frequency when the source and receiver position are not far from the reflection point and the distance between them is less than or equal to a few wavelength [2]. Thus the calculation of sound field in a case of the significant curvature of spherical waves needs the spherical wave reflection coefficient. However it is more complicated and demands much greater CPU-time for calculations. The usage of boundary element methods or finite element methods offers potentially high accuracy but also requires appreciable calculation time [3]. The aim of our work to estimate the expenditure of computational resources to employ spherical wave reflection coefficients for simulating of sound field in rooms.

Theoretical Model

We considered the classical acoustic problem as the reflection of waves radiated by a spherical source above an infinite plane boundary of finite impedance that can be formulated as follows. We wish to find the complex acoustic pressure $p\ (x, y, z)$ at the detector position in Fig.1. which could be found from the equation

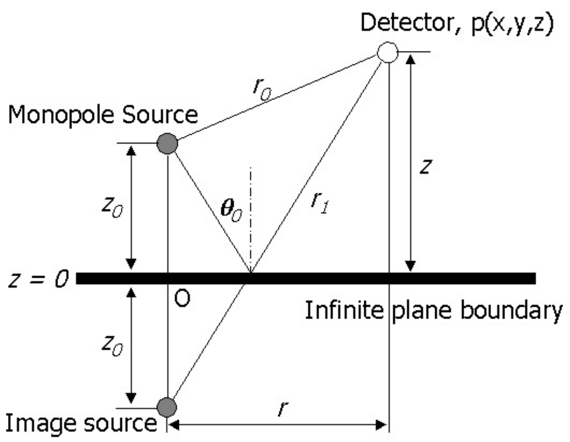

Fig. 1: Configuration of source and detector in a half-space. O is the origin of the coordinate system

for the velocity potential $\Psi(x,y,z)$ in the half-space $z>0$

$$\left(\nabla^2 + k_0^2\right)\psi = \delta(x)\delta(y)\delta(z - z_0) \qquad \text{eq. 1}$$

subjected to the boundary condition

$$\left(\frac{\partial}{\partial z} + ik_0 G\right)\psi \to 0, \qquad z \to 0+$$

$$\psi \to 0, \qquad R = (x^2 + y^2 + z^2)^{1/2} \to \infty$$

where k_0 is the wave number of the spherical wave radiated from and $G=1/Z$ is the normalized acoustical admittance of the locally reacting surface. A common factor $exp\{-j\omega t\}$ has been dropped. The total field above absorber can be write as the sum of a free-space and reflected wave field:

$$\frac{p(x,y,z)}{P_0} = \frac{e^{-jkr_0}}{kr_0} + \frac{p_{refl}}{P_0} \qquad \text{eq. 2}$$

where $P_0=k_0\rho_0/4\pi$ with ambient density of air ρ_0.
By decomposing a spherical wave into an infinite number of plane waves and the angle of incidence into real and imaginary parts by $\theta=\theta'+j\theta''$ the exact form of reflected field [4]

$$\frac{p_{refl}}{P_0} = i\int_0^{\pi/2} J_0(k_0 r\sin\theta')\, e^{-jk_0(z+z_0)\cos\theta'} \times$$

$$\times R(\theta')\sin\theta'\, d\theta' + \int_0^{\infty} J_0(k_0 r\cosh\theta'') \times \qquad \text{eq. 3}$$

$$\times e^{-k_0(z+z_0)\sinh\theta''} R(\frac{\pi}{2} + j\theta'')\cosh\theta''\, d\theta''$$

where $J_0(k_0 r\sin\theta)$ is the Bessel function of the zero order and $R(\theta)$ is the complex reflection coefficient of the infinite plane boundary which depends upon the angle of incident θ and for the plane incident upon it wave can be expressed by

$$R(\theta) = (Z\cos\theta - 1)/(Z\cos\theta + 1)$$

The identity shows that a spherical wave can be decomposed into a summation of product of cylindrical waves in the r direction and a plane wave in z direction.

The formal solution is now complete. However, the most difficult part of the problem is that of obtaining a practical solution one which permits numerical computation. Apparently all of the researches who have addressed the spherical wave propagation over the past century have been faced with this or a very similar integral (eq. 3) and all have necessarily resorted to approximate method for its evaluation. A survey of the literature and of the computational method for the spherical wave propagation over an absorbing plane is given by F.P.Mechel [5]. Those method based on an asymptotic approach, in these, the real axis path of integration is continued into a contour in the complex plane that invariably must be deformed around the branch point and branch cut. In addition, attention is paid to the simple pole, which may or may not be enclosed by the final contour depending on the specific nature of the problem. Theoretical results show good agreement with measurements values but almost all approximations are complicated and sometimes valid only for special cases. Direct numerical integration of the integral (eq.3) seems to be improper and very time consuming. All integrals of the exact forms have oscillating integrands and the interval of integration extends to infinity. But the exponential factor in the second integrand is a rapidly decaying function converging towards zero except grazing incidence. Thus it is mainly the oscillation of the first integrand which makes numerical integration slow. Therefore method of saddle point integration and other method is applied. However now with growth of computer technologies the reduction of a step of integration require essentially smaller expenditure of computational resources. The estimation of CPU-time for direct integration (eq.3) was the task of our work.

Result

We used the numerical technique as the trapezoidal rule. The step of integration was chosen by double recalculation to supply accuracy of 0.1%. The CPU-time and also the percentage value

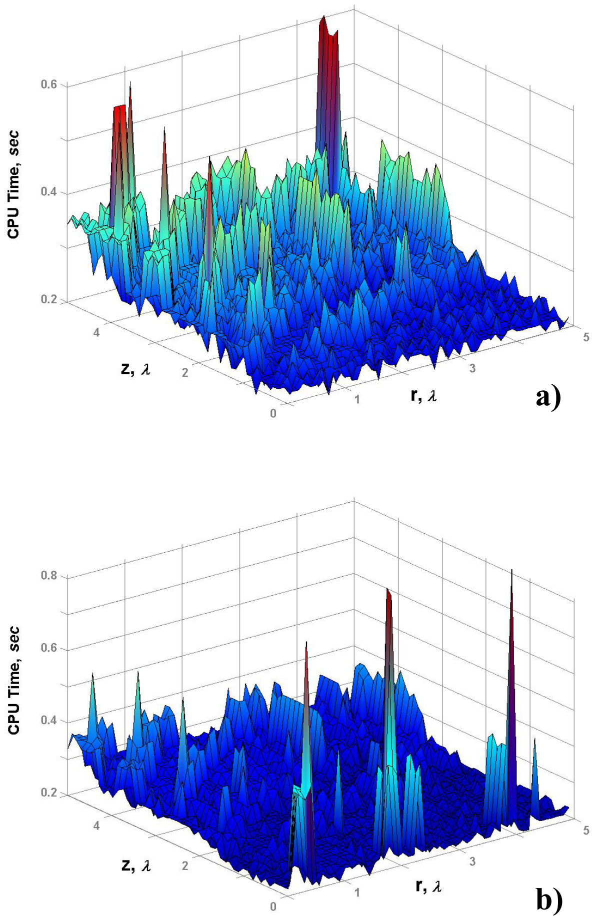

Fig. 2: CPU-Time for various position of receiver at source position in wavelength $z_0 = 1\lambda$ (a) and $z_0 = 0.1\lambda$ (b). Acoustic impedance of absorbing plane $Z = 0.57 - 0.59j$ accordingly diffuse absorption coefficient $\alpha_{diff} = 0.71$

error and phase shift between exact solution and plane wave approximation were calculated at various source and receiver positions for different values of specific acoustic impedance with corresponding to diffuse absorption coefficient. If the source is on distance greater or equal to one more wavelength $z_0 \geq 1\lambda$ then the value error and phase shift are small. The spherical form of a wave has an effect only very close to a surface, so the reflected field could be calculated with great precision by plane reflection coefficient. At such placement of source the angle of incidence is small in comparison with grazing incidence and hence the basic role is played by first integrand (eq.3). It means that convergence of integral is worse far from boundary and consequently CPU-time is greater far from the absorbing surface Fig. 2(a). The diagram shows distribution of CPU-time for various position of receiver, where ordinate and abscissa axis represent the height of receiver above absorbing surface z and accordingly horizontal distance r between source and receiver in wavelength. The average CPU-time on all positions of the receiver is $T_{mean} = 0.26 \pm 0.02$ *sec* (*Pentium IV, 2.4 GHz, 1.5 Gb RAM*). If the source is closer to a surface, the angle of incidence increases and as a consequence the error by plane wave approximation grows. The main role in convergence of integral (eq.3) is transmitted to the second integrand. As a result for very small height of a source above boundary $z_0 \leq 0.1\lambda$ the significant time is expended for calculation of the reflected field closed to a surface Fig. 2(b). The averaged CPU time in that case is $T_{mean} = 0.25 \pm 0.03$ *sec.* Increase of absorption of a surface conduces to a localization of error region between exact solution and plane wave approximation to the source position and to the greater standard deviation of CPU time.

Conclusion and Discussion

The comparative analysis for single reflection problem with plane and spherical wave reflection coefficient was conducted. The result demonstrated that the error at use of plane reflection coefficient is strongly increased at source position $z_0 \leq 1\lambda$ above surface especially for high absorption coefficient. So value error by plane wave approximation reaches $\Delta P > 100\%$ and phase shift $\Delta\varphi > 30^0$. The average calculation time for the spherical reflection coefficient is $T_{mean} < 0.3$ *sec* that makes it's implementation in algorithm of room acoustics simulation for complete volume improper. However the significant error between exact solution and plane wave approximation is occurred in relatively small region, calculation of the reflected field in which leads to considerable improvement of simulated acoustic filed. In this connection additional theoretical researches with more complicated geometry (several surfaces, impedance discontinuity), comparison with predictions of Boundary Element Method and experimental measurements of real models are planned.

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