Numerical simulations of ultrasonic polar scans on single layered fiber reinforced composites
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Introduction
Numerous experiments have already been performed which show qualitatively that ultrasonic polar scans are a promising technique to characterize the stiffness properties of fiber reinforced composites [refs 1-5]. Whereas classical C-scans use normal incident sound on a large number of subsequent spots, a polar scan uses oblique incident sound under each possible angle on one single spot, to obtain information about the material under investigation. Extraction of the stiffness tensor from such polar scans can only occur if there exists a sophisticated model that can actually simulate such scans. Here, we present numerical simulations on single layered fiber reinforced composites as a first step towards complete simulations on multi-layered composites. Characterization of fiber reinforced composites is vital, because their direction dependent stiffness and their value of degradation are most often very critical knowledge during manufacturing and during service.

The principle of an ultrasonic polar scan
Ultrasonic polar scans differ from classical C-scans in that the emitter is placed at a constant distance \( r \) from a particular spot on a composite laminate, with varying angles of incidence \( \theta \) and \( \phi \). The angles vary such that the emitter occupies each possible position on an imaginary half sphere above the plate.

The physics behind an ultrasonic polar scan
If sound impinges a composite laminate, it is scattered, generating sound that enters the laminate, and gets further scattered by the lower interface and then again by the upper interface and so on. During each scattering, sound is emitted in the coupling liquid (water in our case) in which the laminate is swamped. This extremely complicated process is best simulated by considering harmonic incident waves (an incident pulse may always be thought of as being built up by harmonic waves) which give rise to a ‘standing wave phenomenon’ inside the plate. This standing wave pattern is modeled as a superposition of each possible kind of bulk plane wave that fulfils Snell’s law. Such bulk modes are found by demanding plane waves as solutions of Newton’s law of motion [ref. 6]

\[
\frac{\partial^2 \sigma_{ij}}{\partial t^2} = \rho \frac{\partial^2 u_j}{\partial t^2}
\]

and Hooke’s linear law of elasticity [ref. 6]. For composites, the latter reflects orthotropic symmetry, whence it is given by:

\[
\begin{bmatrix}
\sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{13} & \sigma_{23} \\
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{22} & C_{23} & C_{21} & 0 & 0 & 0 \\
C_{33} & C_{32} & C_{31} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{23} \\
2\varepsilon_{13} \\
2\varepsilon_{12}
\end{bmatrix}
\]

Furthermore, continuity of normal stress and normal displacement have to be taken under consideration on the water/plate interfaces. This is given by

\[
u_i^{\text{water}} = u_i^{\text{solid}}, \quad i = 1,2,3
\]

and

\[
\sigma_{i3}^{\text{water}} = \sigma_{i3}^{\text{solid}}, \quad i = 1,2,3
\]

for \( u \) the particle displacement vector and \( \sigma_{ij} \) the stress tensor.

Numerical simulations
As a first stage of advanced numerical simulations, we present the example of a composite laminate which consists of unidirectional...
carbon fibers in an epoxy matrix. The laminate is considered to be 1 mm thick and the applied frequency is 5 MHz. Appropriate damping is taken into account in the plate. In Fig. 2, the numerical simulation of the polar scan for reflection is presented. In Fig. 3, the numerical simulation for transmission is presented. It is seen that particular patterns appear where the reflection or the transmission amplitude becomes negligible. The reason is that for these angles, the generation of quasi Lamb modes occurs, whence sound gets ‘trapped’ before being damped out inside the plate.

In Fig. 2, the numerical simulation of a polar scan of the same specimen, however in transmission. It is seen that the degree of orthotropy also appears in the polar scans. The reason is that the pseudo Lamb waves have velocities that depend on the direction, determined by the stiffness tensor. The faster a Lamb mode, the smaller the angle under which a dark ‘region’ appears in the polar scan.

These characteristic patterns will be used to create an inversion procedure to enable us to determine the complete stiffness tensor of the laminate. In Fig. 4, the numerical simulation of a polar scan in reflection is presented for a plain weave fabric reinforced carbon/epoxy composite. Furthermore, Fig. 5 shows the numerical simulation of a polar scan of the same specimen, however in transmission. The reason is that the pseudo Lamb waves have velocities that depend on the direction, determined by the stiffness tensor.

Concluding Remarks
It is shown by means of numerical examples that polar scans produce characteristic patterns. In the very near future, experiments will be performed that will enable us to use a combined experimental/numerical inversion technique in order to characterize fiber reinforced composites by means of polar scans. The presented theory for single layered composites is about to be extended to multi layered composites. A thorough study will follow as to discover what pseudo Lamb modes cause the patterns and which frequencies are most appropriate for characterization purposes.

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References