Radiation of directional seismic sources within a layered half space

Madjid BERRAKI1,2, Bertrand DUBUS1, Axelle BARONI2
1Institut d’Électronique, de Microélectronique et de Nanotechnologie, département I.S.E.N, U.M.R. C.N.R.S. 8520, 41 Boulevard Vauban, 59046 LILLE CEDEX
2Institut Français du Pétrole, Geophysics department, 1 & 4 Avenue de Bois-Préau, 92852 RUEIL MALMAISON CEDEX
madjid.berraki@isen.fr

Introduction

Oil prospecting requires images of the subsurface. Among measurements that are made to obtain those images, the seismic method consists in studying the propagation time of low frequency (10 - 200 Hz) elastic waves between a source and a receiver, both located at the surface or buried. A recent development in seismic is time lapse or 4D-surveying which aims at monitor the changes in the subsurface in time. To achieve this, measurements are repeated with the same three dimensional configuration: reliability and repeatability of the acquisition footprint are key requirements for 4D surveys.

The weathered zone (WZ) pollutes the signal coming from reservoir variations. This zone is directly located beneath the surface and its mechanical properties vary with weather. To eliminate this perturbation, the proposed solution is a directional source, buried under the weathered zone and radiating downwards.

We present here the modelling of a directional seismic source buried into a layered elastic, homogeneous and isotropic half space. Firstly, Green’s tensor for the half space is derived using the Cagniard De Hoop’s method (CDHM, [1]). Secondly, a source buried in a half space covered by a layer (the WZ) is considered. As the CDHM, would be intricate in this case, the Green’s tensor in a layer is computed using the Discrete Wave Number Method (DWNM, [2]). The solution is then propagated using the reflectivity method [3]. Seismograms are computed for different WZ properties to validate the directional source concept.

I - Directional two point sources in infinite elastic medium

An infinite, elastic, homogeneous and isotropic solid is considered here. Angular frequency, density, speed of L and T waves are respectively denoted ω, ρ, α and β. \(M=(X, Y, Z)\) and \((O, e_x, e_y, e_z)\) and \((O, e_\theta, e_\phi)\) refer to Cartesian and spherical coordinates respectively. A directional seismic source is obtained by combining the couple of forces without moment \(\vec{f}_1(\omega) e_z\) and the force \(\vec{f}_2(\omega) e_x\), separated by a distance \(2d\) (Figure 1a). Adjusting \(\vec{f}_1(\omega)\) and \(\vec{f}_2(\omega)\) such as the radial component of the induced displacement field \(\vec{u}(R, \theta; \omega)\) vanishes along the direction angle \(\theta_0 = 0\), the far field directivity pattern displayed in Figure 1 is obtained.

![Figure 1](image)

(a) : The theoretical directional source
(b) L (full line) and T (dashed line) waves radiated in a whole space \((\alpha^2/\beta^2 = 3, d=0.5 \text{ m}, \text{ frequency} : 100 \text{ Hz})\).

II - Directional source modelling in a semi-infinite medium

II-1 Transient Green’s functions associated to a point force and a couple of forces in a half space

Using the CDHM, the Green’s function \(G = (G_X(M, t), G_Y(M, t), G_Z(M, t))\) associated to a force acting in an elastic, homogeneous and isotropic half space is written as

\[
G(M, t) = \sum_{k=L, T, LT, TT, LL, TL} \frac{\partial}{\partial t} \left[ G''(M, t) \right]
\]  

where \(M=(X, Y, Z)\) is the receiver location and \(G''(M, t)\), the response of the medium to a force having an Heaviside time dependence. The response to a couple of vertical forces is calculated using the representation theorem [4]

\[
G^{\text{Couple}}(M, t) = \frac{\partial}{\partial Z'} G''(M, t) = \frac{\partial^2}{\partial Z'^2} G^{\text{II, Couple}}(M, t)
\]
where $G^{\text{Couple}}(M, t)$ is the response to the couple of vertical forces having an Heaviside time dependence.

**II-2 Source with an arbitrary time dependence**

The time dependence of the source is given by a function $f(t)$. Using the properties of both the Green’s function and the distribution, elastic displacements $u(M, t)$ and $u^{\text{Couple}}(M, t)$ associated to a force and a couple of forces respectively are written as

$$u(M, t) = \sum_{i=1, 7, 11} G^{\text{i}}(M, t) \otimes \frac{\partial}{\partial t} f(t) \quad (3)$$

$$u^{\text{Couple}}(M, t) = G^{\text{H, Couple}}(M, t) \otimes \frac{\partial^2}{\partial t^2} f(t) \quad (4)$$

The directional source model is derived by summing solutions of type (4), computed using a source $S_1$ having a reference time dependence $f_1(t)$, with solutions of type (3), computed for a source $S_2$ having the adjusted time dependence $f_2(t)$.

**II-3 The case of a layered half space**

The modelling of the directional source buried in a half space covered by a layer (the WZ) is now considered. As the CDHM would be intricate in this case, the Green’s tensor in a layer is computed using the DWNM. This method introduces a spatial periodicity of sources to discretize the radiated wave field and relies on the Fourier transform in the frequency domain to calculate the Green’s function. The solution is then propagated in the layered medium using a propagator matrix method [4].

**II - Computed seismograms**

A source buried in a half space made of clay covered by a 15 m thick layer is considered here. Properties of the clay and the layer are $\rho=3300$ kg.m$^{-3}$, $\alpha=2500$ m.s$^{-1}$, $\beta=1443$ m.s$^{-1}$ and $\rho=1610$ kg.m$^{-3}$, $\alpha=500$ m.s$^{-1}$, $\beta=288$ m.s$^{-1}$ respectively. The distance $d$ is equal to 0.5 m and the source depth is set to 20 m. For the directional source and the couple of forces, responses are computed along a vertical line of receiver which horizontal offset from the source is equal to 200 m. Depth of the shallowest receiver is 40 m and depth of the deeper one is 800 m. The time dependence $f_1(t)$ is the Ricker signal with a leading frequency equal to 40 Hz. On the computed seismograms presented below, the horizontal axis represents the time and the vertical axis gives the depth of the receiver.

As the directional source looks like a modified couple of forces, its ability to generate a stable signal can be quantified by comparing the amplitudes of the reflected waves to those induced by the couple of forces. Obtained results are plotted on Figures 2a and 2b : these seismograms represent the reflected part of the horizontal displacement computed for each receiver and normalized by its maximum value in a free space. As expected, the effect of the directional source is to decrease the amplitude of the reflected waves. Between 160 m and 800 m, the latter are completely removed, thus making the radial displacement almost independent of the weathered zone over 160 m; Conclusions are the same for the vertical displacement.

![Figure 2](image)

(a) : seismogram for the couple of forces  
(b) : seismogram for the directional source

**Conclusion**

To ensure repeatability of seismic measurements for monitoring purpose, we aim at design a source which radiation is not affected by the weathered zone. A directional source have been then conceived by combining a force with a couple of forces. Two numerical models have then been developed for studying this source acting in a homogeneous semi-infinite medium or in a layered half space. A typical seismic signal called Ricker has been used to compute seismograms; the obtained results validate the concept of the directional source for 4D seismic applications.

**References**