Measurement of the group velocity of an acoustic wave in an one-dimensional lattice

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Introduction

The aim of this work is to measure the group velocity of an acoustic wave propagating in a complex media. This media is a lattice formed by Helmholtz resonators periodically connected on a cylindrical tube. This kind of system, as it has been demonstrated, presents a strong dispersion characterized by pass and forbidden band in the spectral domain [1].
This property is used to show that the group velocity of an acoustic wave can be higher than sound velocity (acoustical analogy for a group velocity higher than light velocity in electromagnetism).

Propagation in one-dimensional discrete media

A one-dimensional lattice made of an infinite long cylindrical waveguide connected to an array of Helmholtz resonators (numbered by n) is considered. The resonators are connected to the pipe through a pinpoint connection, the radius of the throat cross sectional area s_n of the n^{th} resonator being assumed to be small compared to the wavelength of the acoustic wave. Each connection is located along the axis of the pipe by its coordinate z_n with spacing d_n for two consecutive part as shown in figure 1.

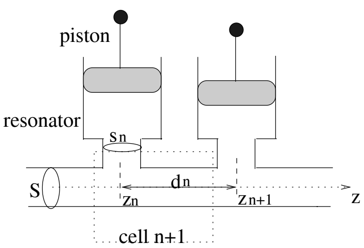

Figure 1: *Dispersive medium used to propagate an acoustic wave.*

The propagation equation of an acoustic wave in such a system is given by [2]:

$$\frac{\partial^2 p(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 p(z,t)}{\partial t^2} = \sum_n \delta(z - z_n)\frac{-\rho s_n}{S}\frac{\partial v_t(z_n,t)}{\partial t} \quad (1)$$

where $p(z,t)$ is the pressure, $v(z,t)$ is the acoustic velocity, ρ is the air density, S is the cross section of the pipe and c is the sound velocity in free space.
The right hand side term acts as an array of secondary point sources (scatterers) which work when they are illuminated by the wave travelling in the pipe.
For a monochromatic acoustic wave whose frequency is below the cut-off frequency of the waveguide, the solution of the equ.1 is seeked by the transfer matrix method. In the $(n+1)^{th}$ cell ($z_n \leq z \leq z_{n+1}$) pressure and velocity are respectively denoted by p_n and v_n, and the solution of $p_n(z)$ is given as a linear combination of wave travelling in opposite direction :

$$p_n(z) = A_n e^{jk(z-z_n)} + B_n e^{-jk(z-z_n)} \quad (2)$$

A_n and B_n being the amplitude of the forward and backward waves.
Propagation through the lattice from z_n to z_{n+m} is described by the relation

$$\begin{bmatrix} A_{n+m} \\ B_{n+m} \end{bmatrix} = \prod_{i=1}^{m} \mathcal{T}_{n+i} \begin{bmatrix} A_n \\ B_n \end{bmatrix} \quad (3)$$

with

$$\mathcal{T}_{n+i} = \begin{bmatrix} (1+\frac{\sigma_{n+i}}{2jk})e^{jkd_{n+i}} & \frac{\sigma_{n+i}}{2jk}e^{-jkd_{n+i}} \\ -\frac{\sigma_{n+i}}{2jk}e^{jkd_{n+i}} & (1-\frac{\sigma_{n+i}}{2jk})e^{-jkd_{n+i}} \end{bmatrix}. \quad (4)$$

For a periodic lattice, the dispersion relation takes the form [2]:

$$\cos(qd) = cos(kd) + \frac{\sigma}{2k}\sin(kd), \quad (5)$$

q being the Bloch wave number and $\sigma = -j\omega\rho s_n/SZ_n$ with Z_n the impedance of each resonator.
Waves that obeys to the relation $|\cos(qd)| \leq 1$ are within a pass band, and travel freely in the duct, and waves such that $|\cos(qd)| > 1$ are in a forbidden band and are quickly spatially damped. They become evanescent so they can not propagate.

Experimental setup

The experimental lattice is a plexiglass tube periodically charged by Helmholtz resonators every $20cm$ as shown in figure 2. The signal acquisition is made by two microphones located at the beginning and at the end of the lattice. Thus, signal stemming from the two microphones can be compared and analyzed. The input signal is a gaussian modulated sinus. Such signal has the particularity to have the same gaussian form in the temporal domain and in the spectral domain.

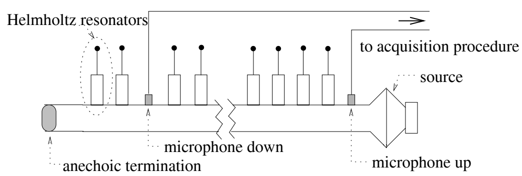

Figure 2: *Dispersive medium used to propagate an acoustic wave.*

Wave velocity has been calculated by two methods based on the comparison of the arrival time difference of input and tunnelled signal. The first method is based on the estimation of the maximum of each signal, and the second one on the estimation of the barycenter of each signal.

Experimental results

A $1D$ lattice act as a filter in the frequency domain. Studying such a medium reveals the presence of forbidden bands and passbands. The studied system presents two kinds of stopbands (illustrated by arrows in figure 3): those due to the resonator include in [300:450]Hz (Helmholtz resonance), and [1100:1200]Hz (corresponding to the length of the cavity), and one other that is the specific characteristic of the periodic lattice ($d_n = 20cm$) at 870Hz.

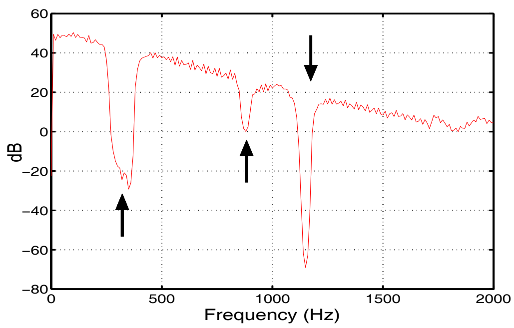

Figure 3: *Frequency response of the periodic lattice.*

Those results are in good agreement with theoretical ones.

Group velocity in a passband

The studied wave frequency is $600Hz$, belonging to a passband. Signal would not suffer from the different interferences at the discontinuities.

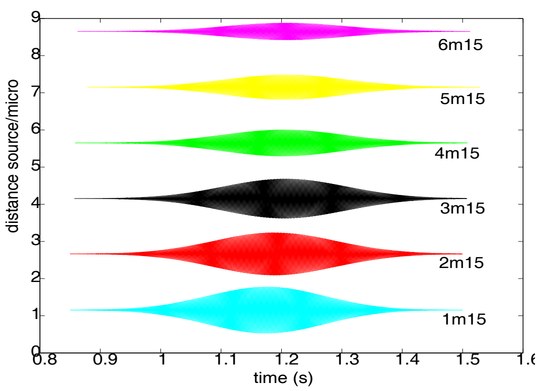

Figure 4: *Temporal signals for different distances from the source in the passband for a* $600Hz$ *wave frequency:* $z = 1.15m, 2.15m, 3.15m, 4.15m, 5.15m, 6.15m$.

Figure 4 shows the temporal signals at different distances from the source. Temporal envelopes are not deformed by the propagation. The only change is the decrease of the wave amplitude, due to the attenuation during the propagation. Results of the wave group velocity calculus are in close agreement with theoretical sound velocity $344m.s^{-1}$.

Group velocity in the Bragg's band

As previously, measurements at different distances from the source have been performed (same location as the previous case). Signal frequency is $885Hz$ belonging to the Bragg's band. The figure 5 shows that, during the

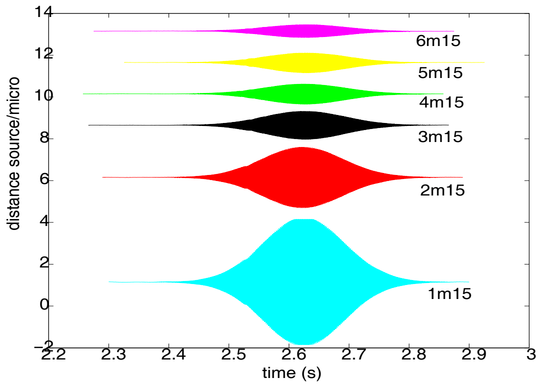

Figure 5: *Temporal signal for different distances from the source in the Bragg's band for a* $885Hz$ *wave frequency:* $z = 1.15m, 2.15m, 3.15m, 4.15m, 5.15m, 6.15m$. *Waves amplitudes have been increased because of the strong attenuation due to the propagation in dispersive medium*

propagation, the signal is not much distorted, but its amplitude is nearly a tenth of amplitude wave belonging to a passband. Wave group velocity is calculated and is estimated to $2000m.s^{-1}$.

Conclusion

It appears that a periodic lattice acts as an anomalous dispersive medium when the frequency of the studied wave is belonging to the Bragg's band. Its group velocity is about $2000m.s^{-1}$, then velocity is then 6 times more important than in free space.
Different researches have been made on this ultra-fast sound velocity phenomenon in some specific propagating conditions but these works are limited to a few number of resonators [1]. The work carried out here allowed to measure propagating velocities higher than sound for 30 resonators. Such a measurement of group velocity higher than sound has never been demonstrated on such distances.

References

[1] C. E. Bradley. Linear and Nonlinear Acoustic Bloch Wave Propagation in Periodic Waveguides. PhD thesis, The University of Texas (1994).

[2] O. Richoux, C. Depollier and J. Hardy. Characterization by a Time-Frequency Method of Classical Waves Propagation in One-Dimensional Lattice: Effects of the Dispersion and Localized Nonlinearities. Acta Acustica united with Acustica. **88** (2002) 934-941.