Acoustical attenuation in a rectangular channel with turbulent flow

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Introduction

Measurements of the attenuation of an acoustical wave in a cylindrical tube in presence of a turbulent flow have been performed by Ronneberger and Ahrens [1], and Peters [2]. The results show different zones that are still not completely explained. Direct Numerical Simulations are needed to understand the problem. We measure the attenuation in a rectangular tube, in order to compare the simulations in a plane channel to experimental results.

Experiments

Experimental set-up

The aim of the experimental apparatus (see Figure 1) is to measure the attenuation in a rectangular tube in the presence of mean flow.

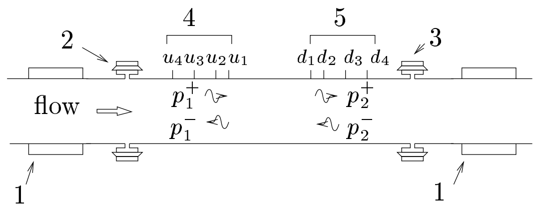

Figure 1: General view of the experimental apparatus. 1: Weakly reflective terminations, 2: upstream source, 3: downstream source, 4: 4 upstream microphones, 5: 4 downstream microphones.

Experiments are carried out for two different states of the system using a "2 sources method": switching on one source while the other is switched off, and conversely, gives results for the sound propagating with or against the flow. Eight microphones are used to overestimate the wavenumbers calculated, and avoid the poor results obtained by the method of transfer functions when the wavelenght is closed to twice the distance between microphones. Weakly reflective terminations on both sides of the measuring device prevent from standing waves.

Measuring technique

The measured pressure at position x is:

$$p(x) = p^+(e^{-ik^+x} + Re^{ik^-x}). \tag{1}$$

The transfer functions H_i between the microphones at the positions $x = x_i$ and $x = 0$, determined during the measurements, are expressed as a function of the reflexion coefficient R which is the ratio of the reflected pressure at x and the incident pressure at $x = 0$:

$$H_i = \frac{p(x_i)}{p(0)} = \frac{e^{-ik^+x} + Re^{ik^-x}}{1+R}. \tag{2}$$

Using another transfer function at $x = x_j$ to get rid of the reflection coefficient gives the following relation:

$$\begin{aligned} F_{ij}(k^+, k^-) = (H_i - e^{-ik^+x_i})(H_j - e^{ik^-x_j}) \\ -(H_i - e^{ik^-x_i})(H_j - e^{-ik^+x_j}) = 0. \end{aligned} \tag{3}$$

Minimizing the F_{ij} for the positions of the microphones availables in the experimental set-up, the values of the wavenumbers k^+ and k^- are calculated. The results should be without flow as follows:

$$k_0 = \frac{\omega}{c_0} + (1-i)\alpha_0 \tag{4}$$

with

$$\alpha_0 = \frac{\omega}{c_0}A \qquad \text{where} \qquad A = \frac{1}{\sqrt{2}Sh}\left(1 + \frac{\gamma - 1}{Pr^{1/2}}\right). \tag{5}$$

The Prandtl number is $Pr = 0.71$ in the air, and the ratio of the specific heats is $\gamma = 1.402$.

With flow ($M \neq 0$), the expressions of the wavenumbers at high frequency are:

$$k_{th}^{\pm} = \frac{(1 + (1-i)A)}{1 \pm (1 + (1-i)A)M}\frac{\omega}{c_0}. \tag{6}$$

Experimental results for wave attenuation

The attenuation is given by the imaginary part of the wavenumbers $\alpha^{\pm} = -I(k^{\pm})$. Results are shown on Figure 2 as a function of the frequency.

The attenuation expected with or without flow, predicted respectively with equations 6 and 4, is smaller than the attenuation measured.

The attenuation without flow (k_0 on Figure 2) is the same whatever the sate of the system (upstream source or downstream source used), and is proportionnal to $\sqrt{f}$ as expected (the shear number $Sh \propto \sqrt{\omega}$, so $\alpha_0 \propto \sqrt{\omega}$).

With flow (k^+ and k^- on Figure 2), symetry and reciprocity are broken. The attenuation for the sound propagating with the flow is higher than against the flow.

For high frequencies, the attenuation increases proportionnally to $\sqrt{f}$, but for small frequencies, the attenuation is constant.

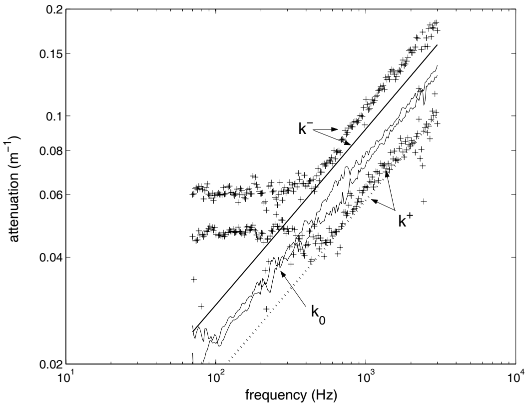

Figure 2: Attenuation $\alpha^{\pm}$ as a function of the frequency (Hz). k_0 measurement without flow. k^+ and k^-, measurement with flow $M = 0.11$ (symbols: measurements, lines: Eq. 6).

The transition between the 2 behaviors at high and small frequencies occurs for a specific value of the coefficient

$$\delta_a^+ = \delta_a.u_\tau/\nu = (\frac{2u_\tau^2}{\nu\omega})^{1/2} \qquad (7)$$

with u_τ the friction velocity. This velocity is calculated for a smooth turbulent flow. Then the friction coefficient is obtained by Blasius formula

$$\lambda = 0.3164(\frac{\nu}{U_0 D_h})^{1/4} \qquad (8)$$

where U_0 is the mean velocity in the tube and D_h is the equivalent hydraulic diameter of the tube. The friction velocity is $u_\tau = \sqrt{\lambda/8}U_0$.

Measurements of the mean attenuation $\bar{\alpha} = (\alpha^+ + \alpha^-)/2$ for 3 different Mach numbers are represented as a function of δ_a^+ on Figure 3.

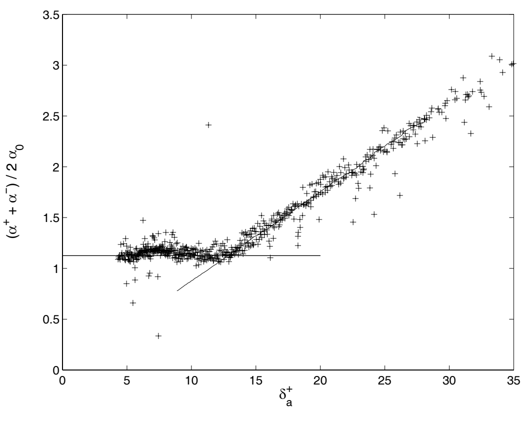

Figure 3: Mean attenuation $\bar{\alpha}/\alpha_0$ for different Mach numbers as a function of the ratio δ_a^+ of the viscous and acoustic sublayers.

Whatever the Mach number, the results coincide and show that δ_a^+ is a pertinent parameter to analyse the measurements. The transition between the 2 behaviors observed occurs at $\delta_a^+ \sim 12$, which was already determined for cylindrical tubes ([1], [2]).

The value ~ 1.125 for high frequencies is related to the discrepancy between measurements of the attenuation $\alpha^{\pm}$ with flow and theory.

If the attenuation, whatever the sens of propagation, is higher than the expected one, the sum $\alpha^+ + \alpha^-$ is higher experimentally, and when devided by $2 \times \alpha_0$, the result is higher than $(\alpha_{th}^+ + \alpha_{th}^-)/(2 \times \alpha_{0th}) = 1$ (with $\alpha_0 \sim \alpha_{0th}$).

If we get rid of the fact that the differences $\alpha^{\pm} - \alpha_0$ are not the same, by calculating an equivalent difference, we still find $\bar{\alpha}/\alpha_0 \sim 1.125$. The explaination for $\bar{\alpha}/\alpha_0 \neq 1$ follows from $\alpha^{\pm}$ greater than the theoretical values. Equation 6 may neglect some effects that lead to $\alpha^{\pm} > \alpha_{th}^{\pm}$.

For small frequencies ($\delta_a^+ > 12$), as α_0 is proportionnal to $\sqrt{f}$ and $\alpha^{\pm}$ are constant according to f, $\bar{\alpha}/\alpha_0$ is proportionnal to $1/\sqrt{f}$. This behavior found on the Figure 3 follows from the behavior of $\alpha^{\pm}$ at small frequencies, which probably comes from the influence of turbulence.

For $\delta_a^+ > 12$, the length of penetration of the wave is important and the turbulence accentuate the attenuation. For high frequencies, the layer in which the shear wave propagates is thin, and seems to be indifferent to the turbulence level. As the model presented can't be used for all the frequencies, a new model taking into account the effect of turbulence should be worked out.

Conclusion

Measurements of the attenuation of sound have been performed in a rectangular tube with and without flow. The discrepancy between predicted and measured values lead us to suppose that the theory doesn't take into account any losses, that would explain why the attenuation measured is more important than expected with the theory. Furthermore, if the attenuation with flow is proportionnal to $\sqrt{f}$ at high frequencies, as it is observed without flow for all the frequency range, at small frequencies ($f < 500Hz$ or $\delta_a^+ > 12$), the attenuation with flow is constant and independant of f. This behavior requires further studies, taking into account the effect of turbulence.

References

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[2] M.C.A.M. Peters, A. Hirschberg, A.J. Reijnen and A.P.J. Wijnands. 1993 *Journal of Fluid Mechanics* **256**, 499–534. Damping and reflection coefficient measurements for an open pipe at low mach and low helmholtz numbers.