Calculation of silencer with circular cross section and centre body

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Application

Silencers with circular cross section are mainly employed to attenuate sound carried by a moderate volume flow without much pressure loss. For larger volume flow, the required larger cross sectional area results in a reduced interaction between the sound field and an absorbing wall, particularly at high frequencies. In order to extend the frequency range of high attenuation, a circular centre body can be employed. Very large ducts are equipped with rectangular splitters, which – however – are not discussed here.

Experience

The performance of silencers is reliably determined from laboratory tests, which are mostly run at room temperature, without flow and at ambient pressure. Many such tests resulted in design charts provided by silencer manufacturers and acoustical consultants. In addition, computer programs have been developed either for interpolation in experimental data or for extrapolation to high temperatures, high flow velocities, pressurized tubes and combinations thereof. Based on the combination of field and laboratory experience with first principles of wave propagation in ducts, software programs are aiming at the optimisation of geometrical and material parameters. Of main interest is the attenuation of the least attenuated mode.

The calculation of this mode has been the subject of many investigations over the last 70 years. In the first place, the decisive transcendental equation for the propagation constant γ as a function of the wall impedance W was solved by graphical presentations of the inverse problem $W(\gamma)$. Later on, explicit approximations have been derived. But calculations aiming at a higher precision or a larger flexibility in parameter selection are often limited by numerical problems.

Special features of circular silencers

While many aspects are common with parallel baffle rectangular silencers, circular silencers need special consideration of

- Bessel functions, which are somewhat more complicated than trigonometric functions,
- the non-symmetrical lining of the outside wall of the duct and the inside wall of a centre body,
- the potential of substantial low-frequency attenuation by a thick lining of the outside wall, and
- the potential of considerable high-frequency attenuation by a an absorbent centre body.

The wave equation in a circular flow duct results in the differential equation for the radial pressure dependence [1, 2]:

$$\frac{1}{k^2}\frac{\mathrm{d}^2 p}{\mathrm{d}r^2}+\frac{\mathrm{d}p}{k\,\mathrm{d}r}\left(\frac{1}{kr}-\frac{2k_z/k}{1-Mk_z/k}\frac{\mathrm{d}M}{k\,\mathrm{d}r}\right)+ p\left[(1-M\,k_z/k)^2-(k_z/k)^2\right]=0 \tag{1}$$

where p is the sound pressure, M is the Mach number of the flow, k is the wave-number in free space and k_z is the axial wave-number in the duct. A systematic approach for the solution under boundary conditions

$$W_i=-\frac{p}{v_r}\bigg|_{r_i} \quad \text{and} \quad W_a=\frac{p}{v_r}\bigg|_{r_a} \tag{2}$$

at an inner and an outer wall, respectively, and for flow with a profile

$$M(r)=1{,}08\,\overline{M}\left(\frac{2}{r_a-r_i}\right)^{\frac{2}{7}}(r_a-r)^{\frac{1}{7}}(r-r_i)^{\frac{1}{7}}\,, \tag{3}$$

which is suitable for engineering applications, has not been published so far.

6-point finite difference scheme

The radial pressure distribution is described by the pressures p_1 to p_4 at 4 points in the free duct and by pressures p_0 and p_5 inside the linings. At point 1, the differential equation (1) is approximated by the difference equation

$$\left[\frac{4}{k(r_a-r_i)}\right]^2(p_0-2p_1+p_2)+ \frac{2}{k(r_a-r_i)}(p_2-p_0)\left(\frac{1}{k\left[r_i+\frac{1}{8}(r_a-r_i)\right]}-\frac{2k_z/k}{1-M_1k_z/k}\frac{(\mathrm{d}M/\mathrm{d}r)_1}{k(r_a-r_i)}\right) + p_1\left[(1-M_1k_z/k)^2-(k_z/k)^2\right]\approx 0. \tag{4}$$

Similar approximations hold for the points 2 to 4. The pressure p_0 outside the free duct is related to the pressure p_1 by

$$\frac{4}{k(r_a-r_i)}(p_1-p_0)=-j\,\rho_0 c\,v_r(r_i)(1-M_ik_z/k). \tag{5}$$

and by the boundary condition

$$Y_i=\frac{p_1-p_0}{p_1+p_0}=\frac{jk(r_a-r_i)\,\rho_0 c}{8\,W_i}. \tag{6}$$

For a bulk reacting lining, W_i depends on the axial wave-number. In an iterative procedure, this dependence is neglected in the first step and taken into account later on. Similar relations hold for the pressure p_5. Introducing the variable

$$x = -2 + \left[\frac{k(r_a - r_i)}{4}\right]^2 \left[\left(1 - \overline{M}\, k_z / k\right)^2 - (k_z / k)^2\right] \qquad (7)$$

an eigen-value problem can be defined by the matrix

$$\begin{bmatrix} x+b_0 & 1+b_1 & 0 & 0 \\ 1-b_2 & x & 1+b_2 & 0 \\ 0 & 1-b_3 & x & 1+b_3 \\ 0 & 0 & 1-b_4 & x+b_5 \end{bmatrix} = 0, \qquad (8)$$

where the coefficient

$$b_1 = \frac{r_a - r_i}{7 r_i + r_a} - \frac{1}{4}\frac{k_z / k}{1 - M_1 k_z / k}\left(\frac{\mathrm{d}M}{\mathrm{d}r}\right)_1 \qquad (9)$$

mainly depends upon the geometry and to a minor extent on the flow distribution and the axial wave-number. For the first step of an iterative procedure, the axial wave-number is approximated by its free-field value.

Similar relations hold for the coefficients b_2 to b_4. The coefficient b_0 includes the boundary condition at the inner lining

$$b_0 = (1 - b_1)\frac{1 - Y_i}{1 + Y_i} \qquad (10)$$

and b_5 the corresponding one at the outer lining. From eq.(8) results a quartic equation in x, which can be solved in closed form. An appropriate algorithm has to be applied in order to select the solution describing the least attenuation or solutions for higher order modes. The result x_0 from a first iterative step yields an improved solution for the axial propagation constant as

$$\left(\frac{k_z}{k}\right) \approx -\overline{M} + \sqrt{1 - \frac{x_0 + 2}{\left[k(r_a - r_i)/4\right]^2}} \;. \qquad (11)$$

In general, the procedure is very robust. However, numerical problems may arise under certain conditions. One of them refers to the case of a pressurized tube. If common absorbent material is specified with a specific flow resistance in the range of 10 kNs/m^4, one should not be surprised to see a major effect of lateral coupling in the lining, since the ratio of the flow resistance to the characteristic impedance ρc may be relatively small. Under such a condition – which is generally not useful and should be avoided for a broad-band absorbent silencer– the first step of the iterative calculation procedure requires more detailed consideration.

Another problem may arise from the application of the procedure to very wide ducts. Bessel functions of large arguments and differences thereof may result in numerical problems. They can be avoided by neglecting the curvature of radial wave fronts.

Examples

The example given in Table 1 refers to results from different software programs, which are consistent for room temperature, but not quite for an elevated temperature. The example described in Table 2 refers to the comparison with measurements on a silencer with rather dense absorbing material [3].

Table 1: Silencer without centre body

Spec. gas const., Nm/kgK	287		Pressure loss	
Temperature, °C; t_{Gas}	500		1,2	Pa / m
Pressure, kPa	100			
Mass flow rate, kg/s	0,55	V, m/s	32,4	
	d_0	d_1	d_2	d_3
Diametre, mm	0	0	219	680
Partition, mm			500	
Flow resistance, kN s m-4				10
Top layer				
flow resistance, kN s m-3				0,1
mass, g/m2		0		0

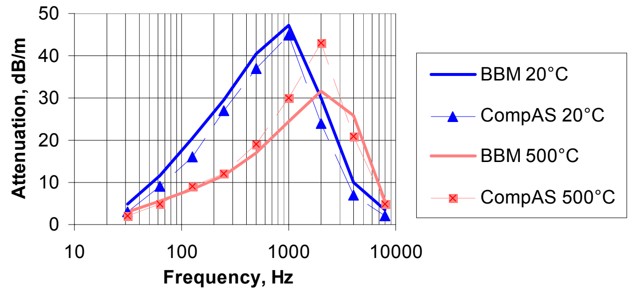
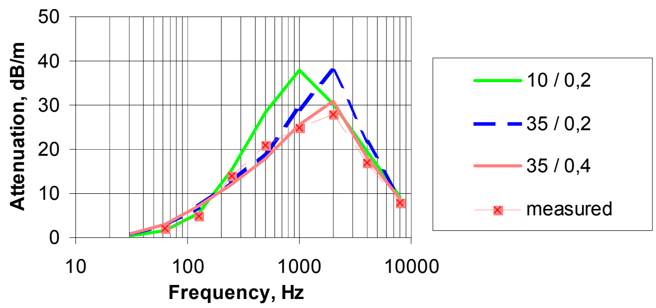


Table 2: Silencer with centre body

Spec. gas const., Nm/kgK		287		Pressure loss	
Temperature, °C; t_{Gas}		20		0	Pa / m
Pressure, kPa		100			
Mass flow rate, kg/s		0	V, m/s	0	
		d_0	d_1	d_2	d_3
Diametre, mm		0	280	500	700
Partition, mm				500	
Flow resistance,	kN s m-4	10 / 35			10 / 35
Top layer					
flow resistance, kN s m-3			0,2 / 0,4		0,2 / 0,4
mass, g/m2			0		0



References

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