Statistical analysis of N-waves characteristics after propagation in turbulent media

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Introduction
Supersonic flight of aircrafts through the atmosphere creates a shock wave called "sonic boom". Due to non-linear effects this wave takes the form of an N, but sonic boom measurements show that the waveforms are distorted during propagation through the atmospheric turbulent layer near the ground [1]. Distortion of N-waves is caused by random inhomogeneities of velocity and temperature. Some authors tried to model the effect of turbulence on sonic boom, but quantitative comparison of sonic boom recordings with theoretical predictions is limited because the parameters of turbulence are usually not measured enough accurately. Therefore, as shown by Lipkens and Blackstock [2], laboratory scale experiments using a downscaled turbulent atmosphere and N-waves produced by electrical sparks offer an attractive alternative since both the acoustic source and the turbulence characteristics can be controlled. The aim of this paper is to present some new experimental results measured with such model experiments.

Experimental set-up
An electrical spark source is used to generate acoustic N-waves. Pressure signals are measured by 1/8” Bruel and Kjaer microphones mounted in a baffle. The main parameters which characterize N-waves are the maximum peak overpressure $P_{\text{max}}$, the rise time $\tau$ defined as the time corresponding to increase of the pressure from 0.1 $P_{\text{max}}$ to 0.9 $P_{\text{max}}$, the wave half duration $T$, and the arrival time $\tau_{\text{arr}}$. Using two set-ups, the acoustic waves are measured after propagation through turbulence in order to investigate the effect of turbulence on these parameters. For each source to microphone distance and each turbulence setting, 100 snapshots are recorded without turbulence and 1000 snapshots with turbulence.

A first set-up (figure 1.a) uses a plane free jet to generate a field of random fluctuations of velocity. The distance between the source and the receivers is 1.4 m; the propagation distance through the width of the jet is 1 m. The mean velocity $U$ of the jet vary from 7 to 15 m/s; the propagation distance through the width of the jet is 1 m. The RMS velocity fluctuations $v_{\text{RMS}}$ along the propagation path of acoustic waves vary from 1.1 to 2.4 m/s. A second set-up (figure 1.b) is based on a heated grid which generates a field of random fluctuations of temperature. The distance of propagation varies from 60 to 450 cm in this case. The RMS temperature fluctuation level $T_{\text{RMS}}$ is 1.4 K for a mean temperature of 310 K. In both cases the integral length scale is of the order of 10 cm, and the spectrum exhibits a decrease following a -5/3 power law in the two decades inertial range, which is characteristic of a fully developed turbulence. Table I compares the parameters of the model experiments to those of the real sonic boom in the atmosphere. The scaling factor is in the range 1/6000 to 1/1000 except for the pressure. Because dissipation in air increases with the frequency, the overpressure cannot be scaled with the same factor since the pressure must be sufficiently high to get non linear effects and create a shockwave.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sonic boom</th>
<th>Model experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak pressure $P_{\text{max}}$</td>
<td>10-600 Pa</td>
<td>10-600 Pa</td>
</tr>
<tr>
<td>rise time $\tau$</td>
<td>0.5-10 ms</td>
<td>2-10 µs</td>
</tr>
<tr>
<td>Duration $T$</td>
<td>80-300 ms</td>
<td>25-45 µs</td>
</tr>
<tr>
<td>$v_{\text{RMS}}$</td>
<td>1.4 K</td>
<td>1.4 K</td>
</tr>
<tr>
<td>$T_{\text{RMS}}$</td>
<td>1.4 K</td>
<td>1.4 K</td>
</tr>
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</table>

Table I: Typical parameters in the case of sonic boom propagation in the atmosphere and in the case of the model experiment.

Results
Turbulence induces fluctuations of the arrival time around the value for obtained in the case of a steady atmosphere. The arrival times fluctuations follow a Gaussian distribution.

The statistical mean peak pressure value computed over the data decreases with the increase of the level of the fluctuations and with the increase of the propagation distance. Nevertheless, the decrease of the mean peak pressure is moderate. In the case of the propagation through a turbulent jet (kinematic turbulence), the mean peak pressure $\langle P_{\text{max}} \rangle = 130$ Pa when $v_{\text{RMS}} = 2.4$ m/s, while it is
$P_{\text{max}} = 150$ Pa for the same propagation distance without turbulence ($v_{\text{RMS}} = 0$ m/s). The distribution of the data around the mean value is not symmetric, the skewness increases from 4.2 to 7.4 when $v_{\text{RMS}}$ increases from 1.1 to 2.4 m/s, and the median of the distribution is lower than the mean value. In many cases the overpressure is slightly attenuated, but in few cases it is strongly reinforced due to focusing at caustics. Peak pressures up to 5 times the value recorded without turbulence were observed. When $v_{\text{RMS}} = 2.4$ m/s, the probability to get peak pressures higher than without turbulence is 30%, the probability to get peak pressures higher than two times the value without turbulence is 1%. In the case of the propagation above a heated grid (thermal turbulence), the microphones are moved away from the source from 60 cm to 450 cm above a grid of resistors. The dispersion of the data around the mean value increases with the propagation distance. The increase of the dispersion starts after about 1.2 m of propagation, which corresponds to the location of the maximum of probability to get the first caustic [3]. The distribution of the data around the mean value is not symmetric and the skewness increases from 0 to 1.7 when the propagation distance increases from 60 to 420 cm. Similarly to the previous experiment, the median of the distribution is lower than the mean value, and the probability to get attenuated waveforms is higher than the probability to get very high peak pressures.

The effect of turbulence on the rise time is significant too. When the waveform is much distorted and has multiple peaks, the rise time has been re-defined as the time required for the pressure to increase from 10% to 90% of the first peak even if it is not the maximum one (this definition is consistent with the results of reference [4] on the loudness of sonic boom). Turbulence increases the mean value of the rise time. The dispersion of the rise time distribution increases both with the level of the fluctuations and the distance of propagation. With turbulence the probability to get rise times shorter than without turbulence decreases when the level of the fluctuations increases. In the case of the propagation through a turbulent jet, when $v_{\text{RMS}} = 1.1$ m/s the probability to get rise times smaller than without turbulence is 15%, while it is only 0.2% when $v_{\text{RMS}} = 2.4$ m/s. Most waveforms with smaller rise time than without turbulence have increased maximum peak pressure. The correlation between increased overpressures and shortened rise times can be attributed to focusing effects at caustics.

The analysis of the number of peaks between the start of the rising of the pressure and the first zero crossing has also been done. The distance for which the number of waveforms with two peaks starts to increase (≈1.2 m) corresponds to the distance for which the probability to meet the first caustic is the highest [3]. Starting from this distance, the mean rise time also increases (figure 2). This behaviour is consistent with the numerical simulations of reference [5] for a configuration close to the “thermal turbulence” experiment. This behaviour is also consistent with the model of Pierce, who explains the increase of the rise time in terms of wavefront folding at caustics [6]. In the case of the sonic boom, Plotkin and George argued that, for long distance propagation, the rise time could reach an equilibrium value resulting form the balance between the scattering process and non linear steepening. The behaviour observed in our experiments shows better agreement with Pierce’s model since the rise time increases as the distance of propagation increases (figure 2) and does not reach an equilibrium value. Nevertheless, one should note that for long propagation distances dissipative effects dominate non linear effects in the model experiment, while it is not the case for full-scale sonic boom.

![Figure 2: Increase of the mean rise time with the distance of propagation (thermal turbulence). —— without turbulence, ---- with turbulence.](image)

**Conclusion**

Turbulence increases the rise time of $N$-waves and decreases the mean of their maximum peak pressures. For propagation distances longer than the distance corresponding to the formation of the first caustic, strongly increased peak pressures can be observed. The transposition of the observations on the scale experiments to the problem of the sonic boom is not straightforward because the scale factor is not exactly the same for all the parameters, nevertheless scale experiments clearly outline that the possibility of focusing by turbulence must be taken into account in order to predict the maximum annoyance due to sonic boom.

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**References**


