Secondary sonic boom modelling for realistic atmospheric conditions

Laurent Dallois[1], Philippe Blanc-Benon[1], Julian Scott[1]
Uwe Berger[2]
David Allwright[3], Katarina Kaouri[3]
[1] Ecole Centrale de Lyon, av. Guy de Collongue, 69134 Ecully Cedex, France
[2] IAP, Schlossstrasse 6, 18225 Kühlungsborn, Germany
[3] OCIAM, Mathematical Institute, 24-29 St Giles', Oxford, OX1 3LB, UK

Introduction

The atmospheric sound speed profile creates waveguides for the shock waves generated by a supersonic aeroplane. The upward shock waves is reflected back to the ground by the temperature gradients in the stratopause or in the thermosphere. The resulting noise disturbance is named secondary sonic boom. It is merely an infrasonic signal and it sounds like a rumble noise associated to bursts [1, 2].

In the present work, the propagation of secondary sonic booms is studied using realistic atmospheric models up to the thermosphere. The secondary carpet position is investigated by solving temporal ray equations. An amplitude equation including nonlinearity, absorption and relaxation by various chemical species is coupled to the ray solver to get the secondary boom signature at the ground level.

The predicted signatures are compared to recorded signals of secondary sonic booms. A good agreement is found for the amplitude and for the duration. The bursts seem to be related to multipath arrivals from direct and indirect secondary sonic boom. The rumbling noise can be interpreted as the effect of finer structures of the atmosphere as gravity waves.

Prediction method

The prediction method is based at least on two assumptions. The first one is that the atmosphere varies over length scales greater than the actual length of the supersonic boom. The second assumption is that the shocks are only weak shocks, *i.e.* shock pressure amplitude is less than a few percent of the underlying pressure field. The model is derived from the generalized Navier-Stokes equations including earth rotation and from the state equations for the different molecular species of the atmosphere to get relaxation effects.

From these equations, a two-step asymptotic development can be conducted. At the first step, a ODE system of six equations is obtained. This system provides the shock wave trajectory called boom rays in this work. The boom rays start at the aeroplane position at a given time τ and are parametrized by their launching angle θ, the azimuthal angle around the aeroplane. The boom rays are quite similar to the acoustical rays due to the weak shock assumption. The main difference is that they are constraint to remain on a surface as the launching polar angle ϕ is fixed ($\phi = \arcsin(1/M)$ where M is the Mach number). At a given time, all the shock waves generated by the aeroplane along its trajectory are located on a surface called the Mach surface.

The second step of the development leads to an amplitude equation which must be solved along each ray. This equation is used to modelized the deformation of the shock wave during its propagation into the inhomogeneous atmosphere. It is a nonlinear paraxial equation which includes dispersion, absorption and relaxation effects. This equation is no more valid when the associated ray goes through a caustic and a Hilbert transform is then applied to the shock wave to simulate the crossing of the caustic.

Atmospheric data

The model presented in the previous section has been applied to realistic atmospheric conditions. The atmospheric fields are provided by the IAP (Leibniz-Institut für Atmosphärenphysik, Universität Rostock). They include mean pressure, density, temperature and wind data at a given latitude ($69°N$) up to an altitude of 150 km. They are discretized every 10 km along the latitude and every 100 m along the altitude.

In addition to these mean values, the IAP provides also simulating data of atmospheric gravity waves. They correspond to finer length scale inhomogeneities of the atmosphere and their influence is investigated by adding them to the mean atmospheric fields.

These data have to be interpolated over the whole propagation domain. This is performed by using third-order polynomials as continuity has to be preserved up to the second spatial derivatives of the fields.

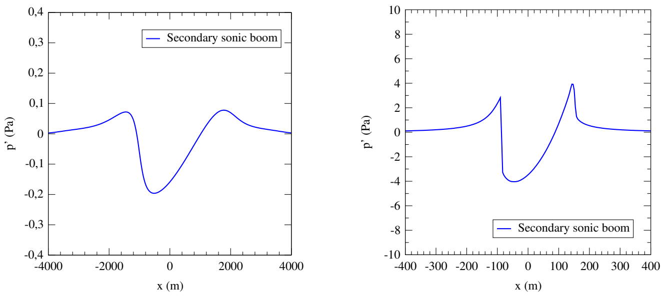
Secondary boom signatures

The annoyances of sonic booms are concentrated at the carpet position where the Mach surface is reflected by the ground. The primary carpet due to the primary sonic booms which travel directly downward the ground are unconcerned by the gravity waves. The secondary sonic booms which travel in the upper part of the atmosphere before being reflected back to the ground are more dependent on the atmospheric structure.

Considering the mean atmospheric sound speed, two waveguides exist for the boom rays. The thermosphere is the upper boundary of the first waveguide. Most of the boom rays trapped in this waveguide reach the ground. As a large part of the boom ray is at altitudes (over

50 km) where absorption and relaxation are the dominant effects, the secondary boom signature is an infrasonic signal (cf. figure 1(a)). Due to its low frequency, this wave can travel over very long distance in the atmosphere. The peak pressure amplitude is around 0.1 Pa and the frequency is around 0.05 Hz. These values are in accordance with long distance measurements of secondary boom [3, 4].

The second waveguide is between the stratopause and the troposphere. As the maximum sound speed in the stratopause is less than the sound speed at ground level, boom rays trapped in this waveguide reach the ground only if the shock wave is also convected by wind at the reflection altitude (around 50 km). The resulting secondary sonic boom signature has a higher amplitude than the previous one with a peak pressure amplitude of 4 Pa (cf. figure 1(b)). Its frequency is also much higher (0.5 Hz) and shocks remain. These values are also in accordance with some measurements of secondary sonic boom [5].



(a) Thermospheric ray (b) Stratospheric ray

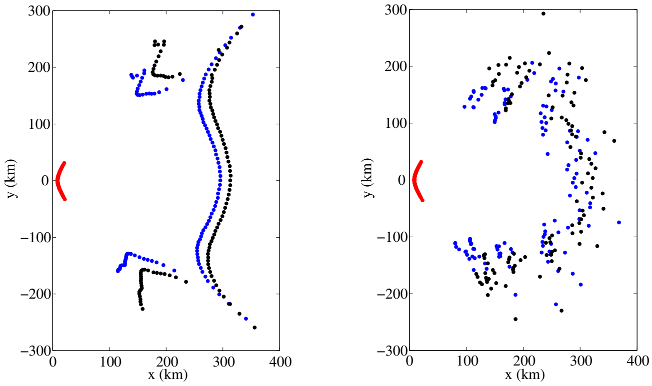
Figure 1: Secondary boom signatures predict by the model.

Gravity wave influence

The primary influence of the gravity waves on the secondary sonic booms concerns the secondary carpets. The figure 2 compares the secondary sonic boom carpets when the atmospheric data do or do not contain gravity waves.

The graph 2(a) is obtained when gravity waves are not included to atmospheric data. The leftmost carpet corresponds to the primary boom. The two carpet patches at a medium distance ($x \sim 150$ km) from the aeroplane are created by boom rays reflected by the stratopause. The last carpets ($x \sim 300$ km) are due to the thermospheric boom rays. Each carpet is composed of a direct and an indirect part, the indirect part being due to the rays reflected back to the atmosphere from the primary carpet position. The distance between the direct and the indirect secondary carpet is around 20 km. This is in accordance with the length time between the bursts of secondary sonic boom (around 30 s). The carpets are smooth and a clear difference appears between each one.

The graph 2(b) is obtained when gravity waves are included. The primary carpet is not influenced by the gravity waves. The gravity wave influence appears on the secondary carpet geometries. The secondary carpets look more complex and the direct and indirect secondary carpets cannot be distinguished anymore. A greater number of rays reaches a given earth location. It may be expected that it is the main reason of the rumble noise.



(a) no gravity waves (b) gravity waves

Figure 2: Secondary boom carpet for realistic atmosphere

Conclusion

The present results show that the signature model predicts well the amplitude and the frequency of the secondary sonic booms. The rumble noise and the bursts may be explained by multipath arrivals of secondary sonic booms. The bursts seems to be linked to the arrival of direct and indirect rays and the gravity waves or other fine scale structures of the atmosphere are responsible for the rumble noise.

Acknowledgements

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