The origin of the audio signal in a beam of modulated ultrasound in air.

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Introduction
The parametric generation of low-frequent sound by means of modulated or interfering ultrasound is well-known in underwater acoustics. The non-linearity of the equation of state produces a low-frequent sound beam (difference frequency of the primaries), which bears the high directivity of the ultrasound radiation. In recent years similar devices were developed to generate audible sound in air. The so-called ultrasound loudspeakers radiate with surprisingly high directivity (“audio spotlight”). However, measurements of the audio-signal with microphones as well as the mere detection with the ear indicate, that the nonlinear generation of the difference frequencies seems to occur not only in the fluid during wave propagation but at the receiver as well.

Theory
With reference to the text book [1] the (1-dimensional) wave equation describing the displacement \( \xi \) and taking the non-linear equation of state into account by parameters A and B (ideal gas: \( B/A = \gamma - 1 \)) reads

\[
\frac{1}{c_0^2} \frac{\partial^2 \xi}{\partial t^2} = \left(1 + \frac{\partial \xi}{\partial x} \right) \frac{\partial^2 \xi}{\partial x^2} \quad (1)
\]

Superposition of two primary waves of different frequencies gives rise to an interference frequency due to the non-linear equation (1). Neglecting viscous loss equation (2) is derived in [2], which is summarized here for two nearly parallel overlaid beams. The amplitude \( p_d \) of the wave of difference frequency \( \Delta \omega \), propagating in same direction of the beams, is proportional to \( \Delta \omega \) and to the pressure \( p_i \) of each primary in the volume of interference. (The directional pattern is omitted here, because it is not evaluated in the experiments).

If the audio production of a modulated primary sound beam is investigated, three primary frequencies interfere and the amplitude \( p_d \) of \( \Delta \omega \) (the audio signal from the modulation frequency) will be proportional to the primary amplitude \( p^3 \), provided the relative modulation depth is kept constant [3].

\[
p_d \propto \frac{V \cdot B}{c^4 \cdot A} \cdot (\Delta \omega)^3 \cdot p_1 \cdot p_2 \quad (2)
\]

\( (V = \text{volume of interference}) \)

Set-up of experiments
In several tests with commercial ultrasound loudspeakers it was observed that the transducers produce a non-linear output at those high levels, which must be used to generate the audio signal. If the distortion of the transducers is in the order of more than 1 %, audio signal components less than 40 dB below the level of the primaries are observed, provided that both ultrasound signals are fed to the same transducer. This distortion signal is easily detected either by measurement of the directional characteristics of the ultrasound loudspeaker or by observing its presence already in front of the ultrasound transducers. Measurements reveal the usual radiation pattern of the ultrasound loudspeaker as if being used for the respective audio frequencies. It must be ensured for such a measurement that no audio sound is generated at the membrane of the microphone, i.e. the ultrasound radiation must be damped sufficiently by appropriate absorbing material which is used to shield the microphone. Evidence was obtained by numerous measurements of directional patterns, but are not reported here.

The set-up used in the present experiments is based on two ultrasound loudspeakers, each being fed by only one of the primary frequencies. Therefore, no distortion signal at difference frequencies is generated at the transducers themselves. The loudspeaker is made from a 5 x 6 array of 400ST ceramic ultrasound transducers [4], which provide each an output of about 115 dB at 30 cm distance at about 40 kHz. The two loudspeakers radiate at the same volume at about 50 cm distance (see Figure 1). The superposed waves are picked up with a tube, pointing at the volume of maximal ultrasound level. A B&K 4135 or a Sennheiser MK4...
microphone are inserted at tube end, both give the same results. By experiment #1 it is proved that no audio signal is generated by the microphone membrane itself or by distortion of the microphone amplifiers. The microphone signal is measured with a B&K 2610 using a band-pass filter to separate the audio signal from the ultrasound. The level of the ultrasound (40 kHz) is 139 dB in front of the tube. The ultrasound level at the microphone at end of the tube is 106 dB. If both speaker radiate with a difference frequency of 9.4 kHz, an audio level of 89 dB is observed. Hence, the audio signals is about 17 dB below the ultrasound, the separation of both is easily obtained by the band-pass. No dependence from temperature was observed (-5°C – 60°C) nor from shape and surface constitution of the tube (i.e. no viscous or “edge”-effects).

**Experiment #1: Location of audio signal production**

With the set-up Figure 1 the level of the generated audio signal (9 kHz) is measured in dependence from the position of a plug of rubber foam. The plug damps the ultrasound much more than the audio signal. The result is plotted in Figure 2. No audio signal (above 50 dB) is produced in front of the tube. The “amount” of audio signal depends on the volume of interaction within the tube, i.e. \( p_d \propto V \) (see eq. (2)).

![Figure 2: Generation of audio signal in a tube in dependence of volume of interaction, which is determined by a plug of rubber foam absorbing the ultrasound waves. Dots: Measurement. Solid line: Eq. (3).](image)

Since \( V \propto d \), the measured dB value depends on \( \log(d) \). The audio level is therefore modelled by eq. (3) with \( L_0 \) = initial level and \( d_0 \) = reference scale.

\[
L_{\text{audio}} \propto \log \left( \frac{d}{d_0} \right) + L_0 \tag{3}
\]

The audio level \( L_{\text{audio}} \) is linear dependent from the each level of the primaries as given by eq. (2) (no figure shown).

**Experiment #2: Dependence from \( \Delta \omega \)**

The audio amplitude \( p_d \) is measured in dependence of \( \Delta \omega \) of the primaries. The level is normalized to the respective ultrasound level. Figure 3 gives the result.

![Figure 3: \( p_d \) as function of \( \Delta \omega \). Solid lines: \( p_d \propto \Delta \omega^2 \) or \( \Delta \omega^3 \). Filled dots: Measurement. Open dots: \( p_d \propto \Delta \omega^3 \) plus tube resonances.](image)

From eq. (2) it is to expect that \( p_d \propto \Delta \omega^2 \). Since the volume in experiment #2 is limited by fixed tube diameter and length, it is assumed that the source strength of the audio wave is determined by this fixed volume of non-linear interaction, which becomes effective with the audio frequency. Therefore \( p_d \propto \Delta \omega^3 \) should be observed. But since the audio wave propagates in a tube of finite length, tube resonances have to be taken into account. Assuming the source at one end of the tube and the microphone at the other, and a magnitude of reflection coefficient of 0.17 assumed at the ends, it is possible to adjust for the increase of sound level at the respective measurement frequencies (cf. filled and open dots in figure 3.).

**Conclusion**

High amplitude ultrasound waves produce an audio signal in air at the difference frequency according to [1]. Viscous effects do not contribute noticeable. Non-linear motion of the receiving transducer (microphone, ear) adds to the audio signal, if the device is hit by the ultrasound beams.

**References**