

Variation in time and spread of quantiles (percentiles) of stochastically fluctuating rms sound pressure

Alois Heiß

Bayerisches Umweltministerium, D-81925 München, Germany, Email: heiss-alois@t-online.de

Introduction

In sound measurement besides the level L_{eq} and other descriptors also the quantile of type "N percent exceedance level", shortly noted "percentile level" or simply "percentile" $L_{x\%}$, is used. Within the measurement techniques standardized until now these quantiles are determined by counting the population of sound pressure level classes. But by this procedure no use is made of statistical structural data additionally embedded within the stochastic fluctuations in time although these data are relevant for the uncertainty of the measurement. But taking into account these structural data is indispensable for quality control in measurements as of percentile levels as of equivalent-continuous sound pressure (mean) levels. For this reason the basic relations governing the uncertainty of quantile measurements, already indicated at other occasions (in [1] for example), here are to be evaluated in an improved, more transparent procedure.

Time dependence of the quantile (percentile)

The time dependence of a quantile, or respectively of a percentile as a quantile rated to 100%, generated by the instant values of a continuous varying quantity, here represented by the sound pressure level (SPL), is implicitly determined by the time dependence of the SPL and by the definition of the percentile as follows:

$$\frac{1}{t} \sum_{i=1}^n w_i(L_q(t)) := \frac{W(L_q(t))}{t} := q = const. \quad (1)$$

$L_q(t)$ denotes the exceedance level corresponding to the excess fraction q , the partition of the signal's instantaneous amplitudes with respect to this exceedance level. The excess fraction - according to [2], section C.2.30 - is predetermined within the admitted range $0 \leq q \leq 1$ by the measurement task and determines the kind of the percentile level to be used.

The basic content of eq. (1) is that at every time t the total W must be equal to $q \cdot t$. This accordingly determines at any time and unambiguously the assigned percentile level $L_q(t)$. From eq. (1) further can be deduced that, for instance, within of an exceedance period (crossing up interval) and progress on a constant level the sum W increases. But to keep eq. (1) valid, the percentile also must drift upward for compensation. This is due to the fact that the total W decreases if the signal sound pressure is getting up. The corresponding is valid for the total of the crossing down intervals u_i , if also q is replaced by the complementary $1-q$. Within a crossing down interval the increase of time by a small step dt causes by eq. (1) an increment of the sum W

$$q \cdot dt = dW = W(L_q(t+dt)) - W(L_q(t)) = \left(\frac{dW}{dL} \right)_{L=L_q} dL_q = -t \cdot \left(\frac{d\Phi(L)}{dL} \right)_{L=L_q} dL_q \equiv -t \cdot \varphi(L_q) dL_q, \quad (2)$$

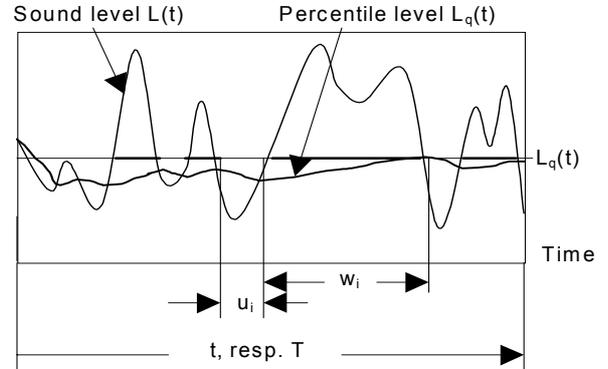


Figure 1: Definition of parameters used in eq. (1). The crossing up intervals w_i are indicated by thick horizontal lines at level $L_q(t)$. The current measurement interval is denoted by t , the final measurement interval by T . The crossing down intervals are denoted by u_i . Their total in reference to T is identical to the cumulative distribution $\Phi(L)$ [3].

with the cumulative distribution $\Phi(L)$ and the probability density function $\varphi(L)$ (p.d.f.) [3]. From eq. (2) follows

$$dL_q = -\frac{q}{t \cdot \varphi(L_q)} dt, \quad (3)$$

the differential equation for the development of L_q in time. After an adequate measurement time t , $u_i/t \ll 1$ becomes valid and the p.d.f. ends to change significantly from one crossing interval to the next. Then the integration of eq. (3) over a crossing down interval u_i results in a continuous sloping down of the percentile level within this interval by

$$\Delta L_{q \downarrow i} = -\frac{q}{t \cdot \varphi(L)} u_i. \quad (4a)$$

The corresponding rise over a crossing up interval is

$$\Delta L_{q \uparrow i} = \frac{1-q}{t \cdot \varphi(L)} w_i. \quad (4b)$$

With increasing measurement time at stationary conditions, i.e. $\varphi(L) \approx const.$, the summing up of the consecutive moving up and down increments eqs. (4a,b) of the percentile (level) converge to zero and consequently the percentile becomes constant in time. Denoting the average duration of a "stochastic period" $\lambda_i := u_i + w_i$ by λ then the expectation value for w_i evidently is determined by $\lambda \cdot q$ and for u_i by $\lambda \cdot (1-q)$ respectively. Taking this into account in the summing up of eqs. (4a) and (4b) for all intervals having occurred, the result is increasing compensation to zero with increasing measurement time.

Confidence interval of the percentile

If fluctuations in time occur randomly the addends in two given by eqs (4a) and (4b) usually variate stochastically independent. Thus the sums of eqs. (4a) and (4b) respectively each are normal distributed (at least in a quite good approximation) according to the Central Limit Theorem [3].

If n stochastic periods occur within the time t the variance of the percentile can be calculated directly by

$$\begin{aligned} \text{Var}(L_q) &= n \cdot [\text{Var}(\Delta L_{q\uparrow i}) + \text{Var}(\Delta L_{q\downarrow i})] \\ &= \frac{n}{t^2 \cdot \varphi(L)^2} \left[(1-q)^2 s_w^2 + q^2 s_u^2 \right] \end{aligned} \quad (5)$$

The finally interesting (symmetric) confidence interval calculates from eq. (5) by Student's factor $t_{n-1;1-\alpha/2}$ with error probability α (α here for example 0,2 \ll 1), confidence level $1-\alpha$ respectively and $n-1$ degrees of freedom [3].

Replacing by practical reasons the notations t , $1/\varphi(L)$, q and $1-q$ respectively by T (as the final duration of the measurement), dL/dq , q_w und q_u and setting $n/T = v$ (as the mean frequency of the stochastic periods), the confidence interval of a percentile, i.e. the distance V_L of the upper and the lower confidence limits L_{qu} resp. L_{ql} from the percentile's value L_q itself can be presented by

$$\begin{aligned} V_L &:= L_{qu} - L_q = L_q - L_{ql} \\ &= t_{n-1;1-\alpha/2} \left| \frac{dL}{dq} \right| \sqrt{\frac{\hat{v}}{T} (q_u^2 s_w^2 + q_w^2 s_u^2)}. \end{aligned} \quad (6)$$

By this formula the confidence interval of percentiles (or quantiles respectively) generated by the instant values of a continuous varying quantity in general can be determined by observable data.

This algorithm is valid for measurements of all kinds of continuous variables, not only in acoustics!

For the online-application of eq. (6) for quality controlled sound pressure percentile level measurement and its extension also to the equivalent-continuous sound pressure level (see below) in acoustical measurement already a corresponding software is available. For details see [④], following the references.

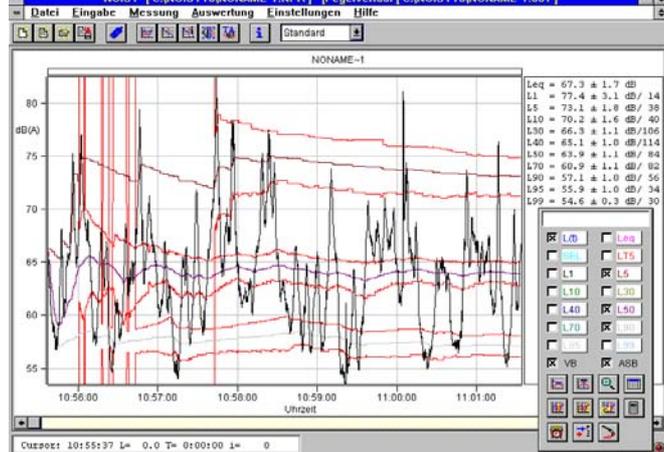


Figure 2 Example of a measurement in practice applying eq. (6). Sound from an suburban main road in 10 m distance. $T = 6$ min. By choice edited are the percentile levels L_5 , L_{50} und L_{90} . The last column in the table on the right above indicates twice the number of crossing intervals which occurred. Further measurement results produced by this technique are reported for example in [4].

The statement of a confidence interval is only meaningful if the - here not explicitly but in [1] cited - confidence intervals of the excess fraction q are positioned within the per-

mitted domain $0 \leq q \leq 1$. From this requirement a minimum number criterion for the stochastic periods to occur can be deduced [1]. For purely random signals this minimum number amounts about 7. The effect of this criterion is documented in Figure 2, especially by the confidence intervals of the edited marginal percentile levels.

Confidence interval of the mean level

The set of all percentiles represent the cumulative distribution function. On this basis and with the established definitions the equivalent-continuous sound pressure level L_{eq} and its confidence limits can be determined. For this purpose the according sound intensity $10^{0,1L_q}$ of the percentiles and of their confidence limits respectively are integrated over $0 \leq q \leq 1$. Finally the result is retransformed into the dB-level space. Condition for this is that the confidence limits of the percentiles are existent, i. e. edited over practically the complete distribution $99\% \geq q \geq 1\%$.

By other authors a procedure was presented in 2003 to get also the confidence limits of $L_{q\%}$ and L_{eq} . The method is to start with one original measurement to get an initial distribution, but without quality monitoring, and then to perform a multiple repetition by computer simulation (bootstrap-method) [5].

References to established regulation of quality control

For the location of the here presented relations and possible procedures within the established regulations it is to be referred exemplary to the guide [2]. The methodology presented above meets the requirements of this guide, primarily concerning the sections 3.1.2, 3.2.1, 3.4.1, 3.4.2, 7.2.3 and 7.4.1.

References

- [1] Heiß, A.: Der Vertrauensbereich des Perzentilpegels (Statistikpegels) bei Echtzeit-Schallmessungen (The Confidence Interval of the Percent Exceedance (Percentile) Level in Online-Measurements; in german language). Fortschritte der Akustik, DAGA '95, Saarbrücken März '95, 679-682.
- [2] DIN V ENV 13005 Guide to the Expression of Uncertainty in Measurement, 1999 - 06.
- [3] Hogg, R.V., Tanis, E.A.: Probability and Statistical Inference. Macmillan New York London (1992).
- [4] Heiß, A.: A Supplement of Sound Measurement Technique by an Online Quality Test Procedure for the Indices L_{eq} and $L_x\%$. J. Sound and Vibr., **241**(1). 29-39 (2001).
- [5] Farelly, F., Brambilla, G.: Determination of Uncertainty in Environmental Noise Measurements by Bootstrap Method. J. Sound and Vibr., **268**(1). 167-175 (2003).

[④] Detailed informations on the measurement software are available from the following address:

WÖLFEL Messsysteme*Software,
Max-Planck-Str. 15, D-97204 Höchberg, GERMANY
Tel. (0931) 49708-500. Email: wms@woelfel.de
URL: <http://www.woelfel.de/wms>