A phenomenological model for active constrained layers

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Introduction

An interesting technique to reduce vibrational energy is Active Constrained Layer (ACL). In the typical ACL configuration, a viscoelastic layer is sandwiched between the structure and a piezoelectric actuator, across which a control voltage is applied.

This paper describes a simple model of beams treated with ACL, which provides insight into the physics of ACL treatments. The equivalent spring stiffness of a free ACL patch subjected to a constant strain at its base is derived. Using a modal approach, this spring is coupled to a mass-spring system modelling the modes of the base beam. Since the strain beneath the ACL patch is assumed to be constant, the model is valid for patches whose length is short compared to the wavelength of the beam, and which are positioned around the antinodes of the beam. The model can be used to study the damping mechanisms of ACL treatments.

Equivalent stiffness of a free ACL patch

The details of the derivation can be found in [1]. An ACL patch of length L_a is subjected to an extensional strain ε_b of constant amplitude at its base. The equivalent stiffness κ_{acl} of the free ACL patch (*i.e.* not bonded to the host structure) is defined as

$$\kappa_{acl} = \frac{F_{shear}}{e_p - e_b}, \tag{1}$$

where $e_b = L_a \epsilon_b$ is the longitudinal extension of the base of the ACL patch, and $e_p = L_a d_{31} V/t_c$, where d_{31} is the strain constant of the piezoelectric actuator and V is the voltage induced across its electrodes. F_{shear} is the shear force applied to the right half of the upper surface of the viscoelastic layer, *i.e.*

$$F_{shear} = b \int_0^{L_a/2} \tau(y) \mathrm{d}y, \tag{2}$$

where b is the width of the beam, τ is the shear stress in the viscoelastic layer, and y is a position variable equal to zero at the center of the ACL patch. Equilibrium of forces in the longitudinal direction yield

$$\tau(y) = \frac{1}{b} f(y) \frac{2}{L_a} \kappa_{acl}(e_p - e_b), \tag{3}$$

$$\text{where} \quad f(y) = \frac{\zeta \sinh \sqrt{g} y}{\cosh(\zeta) - 1} \tag{4}$$

$$\text{and} \quad g = \frac{G}{t_v} \frac{1}{t_c E_c}. \tag{5}$$

In these equations, E_c and t_c are the Young's modulus and the thickness of the cover layer, G and t_v are the shear modulus and the thickness of the viscoelastic layer, and $\zeta = \sqrt{g} L_a/2$. The function $f(y)$ describes the shape of the shear stress τ along the ACL patch, and depends on the non dimensional parameter ζ which contains the characteristics of the ACL patch.

Equations 1, 2 and 3 yield

$$\kappa_{acl} = b \frac{E_c t_c}{L_a} \frac{\cosh(\zeta) - 1}{\cosh(\zeta)}. \tag{6}$$

The lumped parameter model

A beam of length L is treated with an ACL patch positioned between positions $x = x_1$ and $x = x_2$, with its center at $x = x_c$, as shown in Figure 1. A transversal force F_{ext} is applied at $x = x_e$, and a voltage V is applied to the piezoelectric cover layer. Since the mass and

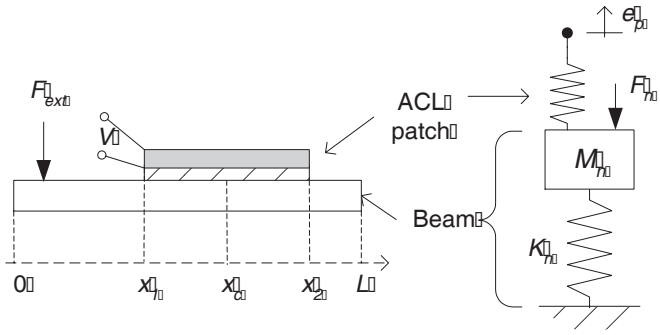

Figure 1: Principle of the lumped parameter model. *Left:* the beam treated with an ACL patch; *right:* lumped parameter modelof a mode of the beam.

the bending stiffness of the ACL patch are neglected, the only effect of the patch is to apply a shear stress τ to the base beam. The equation of motion of the beam is

$$B \frac{\mathrm{d}^4 W}{\mathrm{d}x^4} - \rho \omega^2 W = b \frac{t_b}{2} \frac{\mathrm{d}\tau}{\mathrm{d}x} + F_{ext} \delta(x - x_e), \tag{7}$$

where B, t_c and ρ are the bending stiffness, the thickness and the mass per unit length of the base beam, respectively, W is the transversal deflection, and ω is the angular velocity. If the strain is constant underneath the ACL patch, then the shear stress on the beam is equal to

$$\tau(x) = \frac{1}{b} \frac{2}{L_a} f(x) \kappa_{acl}(e_p - e_b) \Pi(x), \tag{8}$$

$$\text{where} \quad \Pi(x) = 1 \quad \text{if } x \in [x_1, x_2], \quad 0 \quad \text{otherwise.} \tag{9}$$

Following the classical modal decomposition approach, we assume the transversal deflection $W(x,\omega)$ is given by

$$W(x,\omega) = \sum_n W_n(\omega)\Phi_n(x), \tag{10}$$

where $W_n(\omega)$ and $\Phi_n(x)$ are the modal amplitude and the mode shape of the nth mode of the untreated beam, respectively. Inserting 10 and 8 into 7, and neglecting mode coupling effects, yield

$$W_n(K_n - M_n\omega^2) = K_{acl}(e_p - a_n W_n) + F_n. \tag{11}$$

In this equation, K_n, M_n and F_n are the modal stiffness, the modal mass and the modal force of the nth mode, respectively, and

$$a_n = -\frac{t_b}{2}\left(\Psi_n(x_2) - \Psi_n(x_1)\right), \tag{12}$$

where $\Psi_n(x) = \mathrm{d}\Phi_n(x)/\mathrm{d}(x)$. K_{acl} is the equivalent stiffness of the coupled ACL patch and is given by

$$K_{acl} = \frac{t_b}{L_a}\Gamma_n\kappa_{acl}, \tag{13}$$

where κ_{acl} is the equivalent stiffness of the free ACL patch given in 6, and Γ_n is a non dimensional coupling term equal to

$$\Gamma_m = \int_0^L \Phi_m(x)\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x)\Pi(x)\Big)\mathrm{d}x. \tag{14}$$

Equation 11 is the lumped parameter model of the beam treated with ACL, as shown in Figure 1. The left hand side of equation 11 describes a mass-spring system and corresponds to the nth mode of the host beam. The right hand side of the equation describes a spring of modal stiffness K_{acl} and corresponds to the coupled ACL patch. This spring responds to the extension e_p induced in the actuator by the control voltage, and to the extension $a_n W_n$ of the host beam beneath the ACL patch. The non dimensional term a_n corresponds to the coupling between the bending motion and the extensional motion beneath the ACL patch.

ACL damping discussion

In this section, the phenomena associated with ACL damping are discussed and described in terms of the parameters of the lumped parameter model. For the sake of clarity, the subscript n is omitted henceforth. Equation 11 is then rewritten as

$$W(K - M\omega^2) = K_{acl}(e_p - aW) + F. \tag{15}$$

In the passive configuration ($V = 0$), the extension e_p induced in the actuator by the control voltage is zero and hence the effect of the ACL patch is to add to the base beam a stiffness equal to aK_{ACL}. With the assumption that the real part of this stiffness is negligible in comparison with the stiffness K of the base beam, the ACL patch in effect adds a loss factor η_p equal to

$$\eta_p = a\frac{\Im\{K_{acl}\}}{\Re\{K\}}. \tag{16}$$

As expected, the loss factor depends on the coupling a between bending and extension beneath the ACL patch and on the imaginary part of the stiffness of the patch coupled to the beam.

In the case of an active treatment, the material damping increases if e_p, and thus the control voltage, is in phase with $-W$. In this case the extension applied to the spring K_{acl}, and thus the dissipation of energy in this spring, is augmented. If e_p is equal to $-\alpha W$, where α is a positive, real constant, then

$$W = \frac{F}{K - M\omega^2 + K_{acl}\left(\alpha + a\right)}. \tag{17}$$

The total loss factor η is therefore given by $\eta = \eta_p + \eta_a$, where η_a is the loss factor due to the augmented material damping, and is given by

$$\eta_a = \frac{\alpha}{a}\eta_p. \tag{18}$$

In theory, η_a is infinite for infinite values of α, but in practice the amplitude of the control voltage is limited.

Another way of controlling the vibrations is to apply active forces to the host structure through the viscoelastic layer, as can be seen by rewriting equation 15 as

$$W(K - M\omega^2 + K_{acl}) = F + F_a, \tag{19}$$

where F_a is the active force and is equal to

$$F_a = K_{acl}e_p. \tag{20}$$

If $F + F_a = 0$, then the total input power into the structure is zero and so is the response. In practice, complete cancellation cannot be achieved. The control voltage optimising the active forces effect is finite, and decreases as K_{acl} increases. This result suggests that K_{acl} should be chosen as high as possible, *i.e.* the actuator should be bonded directly to the beam; however, this choice would result in a low amount of material damping in the structure, and the beneficial effects of material damping on the stability of the control as well as on the fail-safe characteristics of the treatment would be lost; there is therefore a compromise.

Equation 19 shows that the optimal control voltage is in phase with $-F$. In cases where feedforward control is possible, this phase relationship might be easy to implement, and will usually strongly depend on frequency. In other cases, since at resonance the force is in phase with the velocity of the beam, the active control is essentially a derivative feedforward control.This type of control becomes less efficient away from the resonance, but of course control is of less importance there. An additional beneficial effect of having material damping in the structure is therefore to increase the efficiency of the effect of the active actions in the case of feedback control.

References

[1] K. W. Illaire, H., B. Mace, A phenomenological model of active constrained layers, To be published .