Nonlinear acoustic waves at liquid-solid interfaces

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Introduction

Recent experiments on nonlinear waveform evolution at an interface between a liquid and a solid using laser excitation [1] as well as theoretical work on nonlinear Scholte waves [2, 3, 4] have proven the liquid-solid interface to be an interesting system for the study of nonlinear phenomena of guided acoustic waves. In the following, we shall focus on two aspects of this topic: (i) The description of nonlinear Scholte waves in a situation where the fluid's compressibility is much higher than that of the solid and (ii) waves of predominantly shear-horizontal polarisation in a solid plate immersed in a liquid.

Nonlinear Scholte waves

Acoustic waves at a solid-liquid interface are conveniently described by introducing a displacement field $\mathbf{u}$ both in the solid and the liquid and treating the liquid as a solid with special conditions imposed on its linear and nonlinear elastic moduli [5]. In this and in the following section, we consider wave propagation along the x-direction, while the z-axis is normal to the interface(s), and the displacement field is independent of y and hence depends only on x, z and time t. In view of the recent experiments [1], the solid is supposed to be much less compressible than the liquid. More precisely, we assume that the ratios $\rho_F v_F^2/c_{IJ}$ are of order $O(\nu)$, where c_{IJ} are second-order elastic moduli of the solid, v_F is the sound velocity and ρ_F the mass density of the liquid, and $0 < \nu \ll 1$. Carrying out an asymptotic expansion of the displacement field in powers of ν, $\mathbf{u} = \nu^2\mathbf{u}^{(2)} + \nu^3\mathbf{u}^{(3)} + O(\nu^4)$, introducing the coordinates $\xi = x - v_F t$, $\eta = \nu z$, $\tau = \nu^2 v_F t$, and the partial Fourier transform of the local Mach number in the fluid, $\dot{u}_1^{(2)}(x,z,t)/v_F = \int_{-\infty}^{\infty} e^{iq\xi} B(q,\eta,\tau)\frac{dq}{2\pi}$, one is led to the following evolution equation:

$$ik\left[2\frac{\partial}{\partial\tau}B(k) + \varepsilon\int_{-\infty}^{\infty} B(q)B(k-q)\frac{dq}{2\pi}\right] + \frac{\partial^2}{\partial\eta^2}B(k) = 0, \quad (1)$$

where ε is the nonlinearity parameter of the fluid ($z > 0$), and to the following linear boundary condition at the liquid-solid interface ($\eta = 0$):

$$\left[\frac{\partial}{\partial\eta}B(k,\eta,\tau)\right]_{\eta=0} = -|k|b(k)B(k,0,\tau). \quad (2)$$

It is only in the function $b(k)$ that the properties of the solid enter. If the solid is a homogeneous medium, $b(k) = b_0$ is a constant. (A detailed derivation may be found in Ref. [4].) Equation 1 is the 2-dimensional Zabolotskaya-Khokhlov (ZK) equation [6].

The authors of Ref. [1] have analysed their experimental data on the basis of the inviscid Burgers equation, which may be written in the form

$$\frac{\partial}{\partial\tau}U(k) = -\frac{ik\varepsilon}{2}\left\{\int_0^k U(q)U(k-q)\,\frac{dq}{2\pi}\right.$$

$$\left.+2\int_k^{\infty} f(k/q)U(q)U^*(q-k)\,\frac{dq}{2\pi}\right\} \quad (3)$$

with function $f(\zeta) = 1$ independent of its argument. On the other hand, Meegan et al.[3] derived a 1-dimensional evolution equation for nonlinear Scholte waves which reduces to (3), however with $f(\zeta) = \zeta$, in the case of negligible nonlinearity of the solid. When transforming (3) into real space, the factor $f(k/q) = k/q$ causes the nonlinearity to be strongly nonlocal. At first glance, this strongly nonlocal evolution equation and the inviscid Burgers equation do not seem to refer to the same physical system. However, a more careful analysis reveals that they are 1-dimensional reductions of the 2-dimensional ZK equation (1) with boundary condition (2) in two different limiting cases.

With increasing elastic moduli of the solid, the right-hand side of (2) tends to zero, and one may seek solutions $B(k)$ of (1) and (2) that are independent of η. They obviously have to satisfy (3) with $f = 1$ when B is identified with U. In this limiting case, the waveguiding effect of the interface is neglected in comparison with the nonlinearity in the liquid.

The evolution equation (3) with $f(\zeta) = \zeta$ refers to the opposite limit, where the guiding effect of the interface is dominant. Here, a second asymptotic expansion with parameter $\tilde{\nu}$, $0 < \tilde{\nu} \ll 1$, $B = \tilde{\nu}B^{(1)} + \tilde{\nu}^2B^{(2)} + O(\tilde{\nu}^3)$ may be applied with lowest-order term $B^{(1)}(k,\eta,\tau) = U(k,T)\exp[-k(b_0\eta + i\Delta\tau)]$. $T = \tilde{\nu}\tau$ is another stretched coordinate and $\Delta = -b_0^2/2$. The linear inhomogeneous boundary value problem for $B^{(2)}$ at order $O(\tilde{\nu}^2)$ requires a compatibility condition which is evolution equation (3) with $f(\zeta) = \zeta$ after renaming $T \to \tau$.

If the solid is inhomogeneous, an additional linear dispersion term appears in (3). The latter asymptotic expansion can then be used to construct solitary solutions of (1) and (2) in a way similar to Rayleigh-type solitary waves [7, 4]. The contribution $V^{(1)}$ of the leading-order term of this expansion to the local Mach number V, corresponding to a solitary Scholte wave solution, is shown in Fig. 1 for the special case $b(k) - b_0 \propto k^2$.

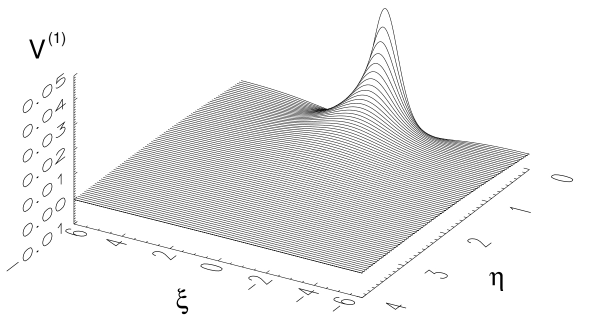

Figure 1: Local Mach number in the liquid, associated with a solitary Scholte wave (internal units).

Nonlinear predominantly shear-horizontal waves in a plate

The lowest branch in the dispersion relation of shear-horizontal plate-modes is non-dispersive. The shear strain associated with the corresponding acoustic waves is homogeneous, and consequently it does not "feel" the boundaries of the plate. This is no longer the case if nonlinearity is taken into account. Shear waves couple to longitudinal components of the displacement field via second-order nonlinearity. In bulk media, this merely leads to a modification of the constant in front of the third-order nonlinear term in the corresponding scalar evolution equation describing the propagation of nonlinear shear waves. In elastic plates, the sagittal components of the displacement field are influenced by the finite thickness of the plate. This gives rise to a nonlinear dispersion term in the evolution equation governing the propagation of predominantly shear-horizontal waves in a plate. If the plate is immersed in a liquid with sound velocity v_F smaller than the velocity v_T of transverse sound waves in the plate, a nonlinear damping term appears in addition, reflecting radiation damping. Using methods mentioned in the previous section, the following evolution equation is obtained for the Fourier transform $B(k,\tau)$ of the leading-order contribution to the shear strain $u_{2,1}(x - v_T t, \tau)$ in an asymptotic expansion:

$$i\frac{\partial}{\partial\tau}B(k) = k\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[C_1 + C_2 M(qH + q'H)]$$

$$\times B(q)B(q')B(k-q-q')\frac{dq}{2\pi}\frac{dq'}{2\pi}. \qquad (4)$$

The plate has been assumed to have thickness $2H$ and to consist of a cubic elastic medium with the crystallographic axes along the coordinate axes. (4) contains the complex function M which has the form

$$M(\zeta) = \frac{\tanh(\alpha\zeta)}{\alpha\zeta - [K - iJ\zeta]\tanh(\alpha\zeta)}. \qquad (5)$$

in the case $v_F < v_T$. C_1, C_2, K, J are real constants, α is either real or imaginary. The imaginary part of M reflects nonlinear damping due to radiation of sound waves into the liquid. In the case $v_F > v_T$, M has a pole corresponding to a nonlinear resonance of the shear-horizontal plate modes with Scholte-Lamb waves.

The nonlinear dispersion term in (4) has been discussed in connection with "exotic solitons" [8]. It has also strong implications on shock formation of shear waves, a phenomenon that has recently been verified experimentally in a bulk solid material [9]. Fig. 2 shows the evolution of an initially sinusoidal waveform with period L for three different values of the parameter $\zeta_0 = 4\pi\alpha H/L$ in an example system. The distance from the source is the same in all three cases. For $\zeta_0 = 8.07$, the efficiency of third-harmonic generation is strongly suppressed in comparison to shear bulk waves ($\zeta_0 = \infty$).

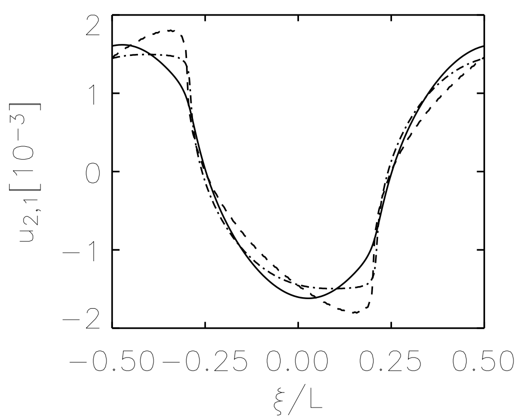

Figure 2: Waveform evolution of nonlinear shear-horizontal waves in an elastic plate (RbCl) immersed in a liquid (CCl_4). $\zeta_0 = 8.07$ (solid), $\zeta_0 = 16.14$ (dashed-dotted), $\zeta_0 = \infty$ (dashed).

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