Elastic nonlinearity of soft solids using transient elastography

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Introduction

If nonlinearity has long been studied in metals, crystals or rocks, almost no experimental works is found in the literature concerning soft tissues. Thanks to the ultrafast scanner, we present in this paper an overview of three transient elastography experiments that quantify the nonlinear behavior of a soft tissue phantom. In the first one, a static stress is applied on a phantom. The change on the shear wave speed characterizes the nonlinear elastic Landau moduli. The surprising difference found between these constants are thought to be closely related to the huge difference between the linear Lamé coefficients ($\lambda >> \mu$). It is the acoustoelasticity experiment. In the second one, we present the first experimental observation of a shock shear wave. The very weak Young's modulus of the tissue phantom allows one to generate plane shear wave with a Mach number as high as unity. In this extreme configuration, the agreement with the theoretical prediction of the modified Burgers equation is remarkable. It is the finite-amplitude shear wave experiment. At last, the interaction between two plane transverse waves with frequencies ω_1 and ω_2 is carefully studied. Harmonics are created during the propagation at the frequencies ($3\omega_1, 3\omega_2, \omega_1+2\omega_2, \omega_1-2\omega_2, \omega_2+2\omega_1, \omega_2-2\omega_1$). It is the parametric interaction experiment. This set of three experiments involves plane transverse waves. The feasibility of measuring the nonlinear elastic coefficient with non plane shear wave is discussed in the last section.

Acoustoelasticity experiment

An Agar-gelatin based phantom is investigated with the transient elastography technique [1]. The typical speed of shear waves at 100 Hz is 2.8 m.s^{-1}. A static uniaxial stress is applied. Experimentally, loads are set on a rigid plate placed on the top of the phantom. Then, one can observe changes on the speed measurements of shear waves polarized in the direction parallel or perpendicular to the stress axis [2,3], Figure. 1. From the slopes, one can found the following values -101 kPa and -4.3 GPa for the Landau coefficients A, B respectively [4]. The huge difference between these third order moduli is striking since in more conventional media such as metal, rocks or crystals they are of the same order.

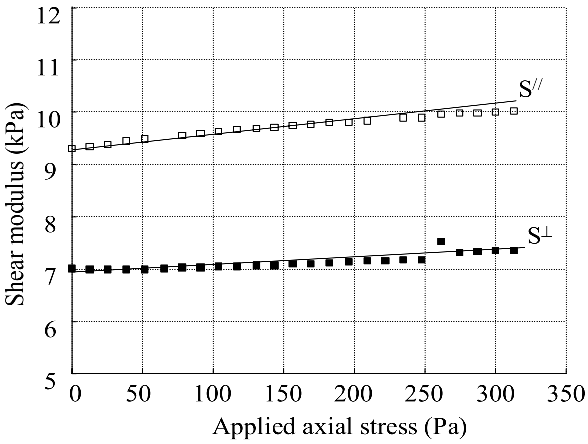

Figure 1 : Experimental shear moduli as function of the uniaxial stress. From the two slopes, the two nonlinear Landau coefficients A and B are computed.

Shock transverse wave

In the field of soft solids, such as biological tissues (muscle, fat, breast) or Agar-gelatin based phantom (a soft tissue model), the very low value of the shear elasticity (typically 2.5 kPa) allows the propagation of a low frequency transverse wave (100 Hz) with a very high particle velocity (0.6 m.s^{-1}) compared to its speed (1.6 m.s^{-1}). Thus Mach numbers as huge as unity are obtained. Consequently in this configuration, third order nonlinear effects become very high and clearly modify the transverse wave shape. The temporal shape of the particle velocity at 15 mm away from the source is not a saw-tooth shape as for the longitudinal waves. As predicted by theory [5], for such plane waves one can observe only odd harmonics at 100 Hz, 300 Hz and 500 Hz (Figure 2(b).). As a comparison, the spectrum of the accelerometer set on the vibrator (the shear wave source) only contains the fundamental harmonic at 100 Hz [6].

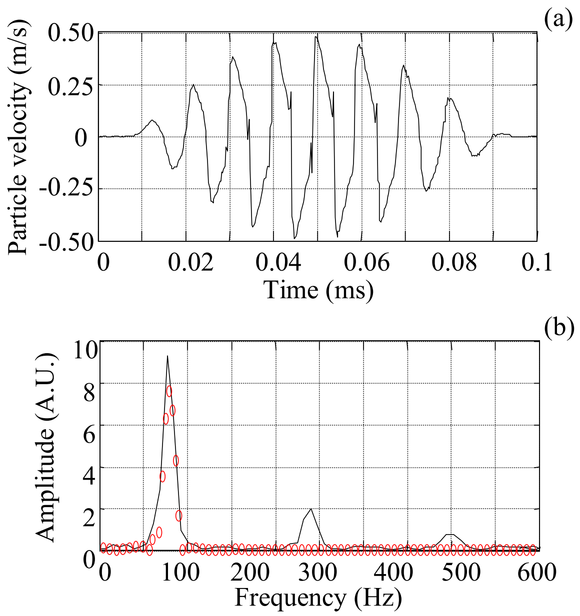

Figure 2 : Experimental particle velocities as function of time measured at a point located at 15 mm away from the source (a) and its normalized spectrum (b). The central frequency of the emitted signal is 100 Hz. Experimental spectrum of the acceleration signal measured on the source (circles).

Nonlinear interaction between transverse wave

As in the case of shock transverse wave, nonlinear interaction between transverse waves is an effect one order smaller than between longitudinal waves. In the latter case, the nonlinear interaction between two primary waves ω_1 and ω_2 (respectively at 100 and 140 Hz) give birth to waves at frequencies $2\omega_1$, $2\omega_2$, $\omega_1+\omega_2$, $\omega_1-\omega_2$. For transverse waves, from the nonlinear wave equation, and using some perturbation method, one show that two primary waves (ω_1 and ω_2) create in first approximation harmonics at the frequencies $3\omega_1$, $3\omega_2$, $\omega_1+2\omega_2$, $\omega_1-2\omega_2$, $\omega_2+2\omega_1$, $\omega_2-2\omega_1$. In both mode (longitudinal or transverse) waves vector have to be collinear in order to give rise to a cumulative effect. This theoretical result is well retrieved in the numerical simulation of the modified Burgers equation as well as in the experiment, Fig.3.

Conclusion

Although gelatin based phantoms are known to be very linear from an elastic point of view, quantitative measurements of their nonlinear behavior is possible with elastographic methods. The experiments presented in this paper are probably still too academic to be easily applied on real soft tissues but give a clear illustration of the shear wave behavior in soft media. Further works are needed to investigate the non linear behavior of non plane shear waves in order to apply these techniques in vivo.

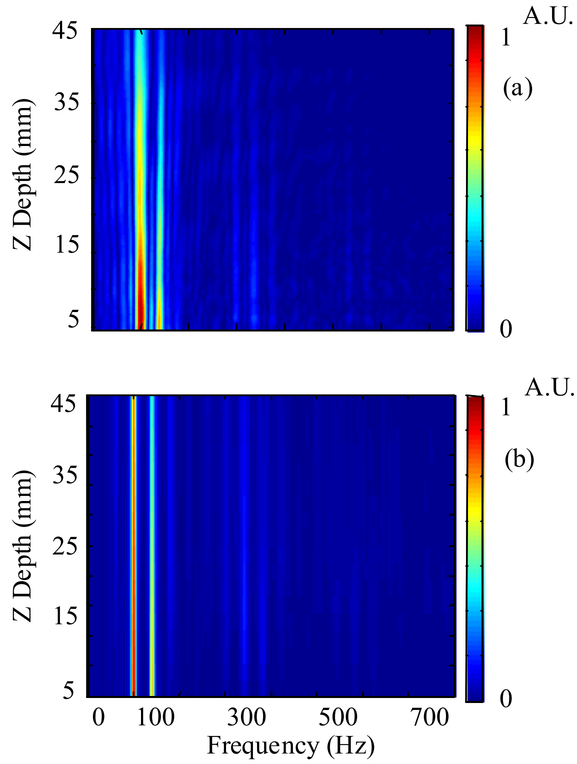

Figure 3 : Experimental (a) and simulated (b) spectrum of the wave profile at each depth when two transverse waves at ω_1 and ω_2 are sent in an Agar-gelatin based phantom.

References

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