Acoustic Simulation of an Idealized Exhaust System by Coupled FEM and Fast Mulitpole BEM

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Introduction

The sound properties of modern cars play an increasingly important role on the overall impression of a vehicle. One of the major sound sources is hereby the exhaust system. Exhaust systems are exposed to large pressure pulsations resulting from the periodically blown out exhaust gas of the piston engine. These large pressure pulsations lead to vibrations of the thin walls of the exhaust system and thus generate structure-borne sound which is additionally contributing to the sound radiation of the exhaust system as well as the sound at the tail pipe outlet. In this paper, the sound radiation of an idealized exhaust system is analyzed. This work focuses on the contribution of structure-borne sound excited by the acoustic path on the overall sound radiation compared to the sound radiation of the tail pipe end.

Simulation model

The simulation model of the idealized exhaust system consists of two sub-models. In the first step, a fully coupled FE-FE-model is used to compute the interior acoustic problem of the exhaust system. The fluid-structure interface accounts for the influence of the acoustics on the flexible walls. With this the excitation of structural vibration modes by the acoustic path is computed. In a next step, the calculated normal velocities on the boundary of the structural parts of the exhaust system are used as boundary data for the BE-model in order to solve the exterior acoustic problem.

Idealized exhaust system: The idealized exhaust system, which is analyzed in this work consists of one expansion chamber, an elbow, an exhaust pipe and a tail pipe as shown in Fig. 1. The system is fixed at its ends by circular soft rings. The system is excited on the inlet with a uniform, harmonic pulsating pressure $p = \hat{p}\mathrm{e}^{\mathrm{j}\omega t}$. This model does not consider mean flow and vortexes. It assumes that the linear acoustics theory is applicable. The total pipe length is 2.4 m, the pipe radius is chosen as $r_0 = 0.034$ m.

Interior acoustic problem

FSI-coupled undamped problems are formulated by FE-discretization as follows [4]

$$\begin{bmatrix} M_\mathrm{s} & 0 \\ M_\mathrm{fs} & M_\mathrm{f} \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} K_\mathrm{s} & K_\mathrm{fs} \\ 0 & K_\mathrm{f} \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} F^\mathrm{u} \\ F^\mathrm{f} \end{bmatrix}, \qquad (1)$$

with the nodal displacement degrees of freedom u, and the nodal acoustic pressure degrees of freedom p. The

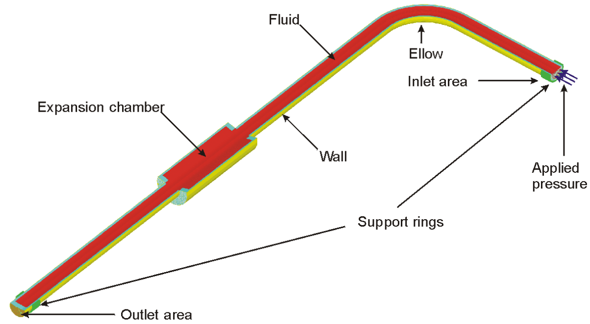

Figure 1: Structure of the idealized exhaust system

mass coupling matrix M_fs and stiffness coupling matrix K_fs are related by $M_\mathrm{fs} = -\rho K_\mathrm{fs}^T$.

At the outlet an impedance condition is applied using the results derived in [2]: The specific acoustic impedance Z can be expressed in terms of the complex reflection coefficient R by

$$Z = \frac{p}{v_n} = \rho c \frac{1+R}{1-R} \quad . \qquad (2)$$

The complex reflection coefficient R can be characterized by its magnitude $|R|$ and the end correction factor δ

$$R = |R| \mathrm{e}^{j(\pi - 2k_0\delta)} \quad . \qquad (3)$$

The values of $|R|$ and δ for the system at hand are shown in Fig. 2. For low frequencies the magnitude of the re-

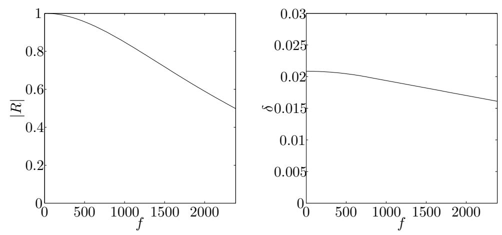

Figure 2: Impedance condition: Magnitude and end correction factor of a circular pipe (radius $r_0 = 0.034$ m).

flection coefficient is close to unity while its phase angle is almost π, hence an incoming wave is almost fully reflected at the tail pipe end. In the FE-program ANSYS the complex impedance condition (2) is modeled in the harmonic case by adding massless surface elements onto the outlet, assigned with viscosity and added mass per unit area.

In the simulation the expansion chamber shows a typical transmission loss behavior with a maximum value of 20 dB [3]. The modal analysis of the in-vacuo system shows resonance frequencies as indicated by the vertical lines in the lower plot of Fig. 3.

Exterior acoustic problem

Having solved the interior acoustic problem, the computed displacements on the pipe and the pressure values on the outlet are used as boundary data for the exterior acoustic problem, yielding values for the pressure and acoustic flux respectively on the surface of the exhaust system. This problem is computed by means of the Fast Multipole Multilevel Boundary Element Method (FMM) [1] enabling a very fast and efficient computation.

Sound radiation: The sound radiation of the exhaust system is evaluated by separately computing the radiated sound power through the cross-section of the outlet W_{Γ_D} and through the walls of the exhaust system W_{Γ_N}

$$W_\Gamma = W_{\Gamma_\mathrm{D}} + W_{\Gamma_\mathrm{N}} = \int_{\Gamma_\mathrm{D}} \boldsymbol{I} \cdot \boldsymbol{n} \mathrm{d}\Gamma + \int_{\Gamma_\mathrm{N}} \boldsymbol{I} \cdot \boldsymbol{n} \mathrm{d}\Gamma \quad , \quad (4)$$

where $\boldsymbol{I}$ is the local intensity vector on the surface and $\boldsymbol{n}$ the unit normal vector pointing outwards from the closed surface $\Gamma = \Gamma_\mathrm{D} + \Gamma_\mathrm{N}$. The solid line in the upper plot of Fig. 3 shows the total radiated sound power W_Γ scaled to its maximum value. The dash-dotted line indicates the magnitude of the sound power radiated by the structure borne sound W_{Γ_N}. The plot clearly shows that W_{Γ_N} does not contribute significantly to the total radiated sound power for the vast majority of frequencies. Yet, for some frequencies marked with an asterisk energy is absorbed by the walls, though the values are very small.

For some frequencies close to the eigen-frequencies of the in-vacuo exhaust system W_{Γ_N} is strongly contributing to the total radiated sound power. Since the FE-model is undamped, the amplitudes are unbounded at resonance frequencies.

Fig. 4 shows the computed sound field for the frequency $f = 254$ Hz. The vertical plane is chosen at a distance of 20 cm from the longitudinal axis of the exhaust system, giving a good impression how the sound pressure looks like at the underfloor of a car.

For the presented idealized exhaust system the simulation results suggest a minor contribution of the structure-borne sound to the total radiated sound power. In the case of a more realistic exhaust system, especially for the case of a more effective muffler with a much higher transmission loss, the influence of the structure-borne sound will play a more important role. In this work only the excitation of structural vibration by the acoustic path is considered. Vibrations induced by the movement of the attached engine are not taken into account. However, the suggested method proofs viable for evaluating the sound radiation of exhaust systems and can easily be extended to account for these additional effects.

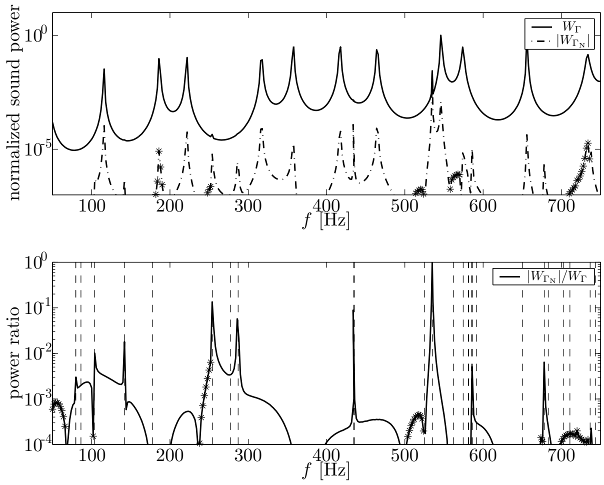

Figure 3: Top: Normalized radiated sound power of the exhaust system W_Γ and the contribution of the walls of exhaust structure $|W_{\Gamma_\mathrm{N}}|$. Bottom: Ratio of the $|W_{\Gamma_\mathrm{N}}|$ to W_Γ. W_{Γ_N} does not contribute significantly to the overall radiated sound power W_Γ, except for the case of pressure induced resonance of the exhaust structure.

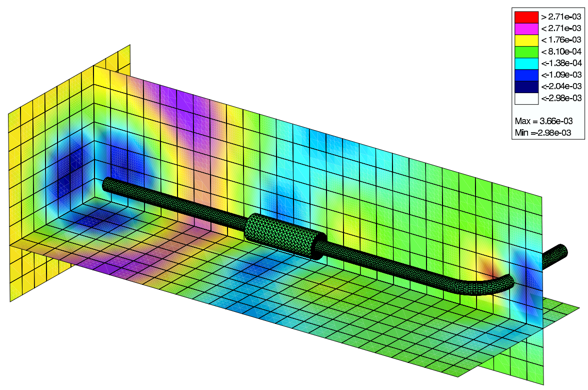

Figure 4: Sound field of vibrating exhaust system at $f = 254$ Hz.

Acknowledgement

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References

[1] M. Fischer. *The Fast Multipole Boundary Element Method and its Applications to Structure-Acoustic Field Interaction.* PhD thesis, Universität Stuttgart, 2004.

[2] H. Levine and J. Schwinger. On the radiation of sound from an unflanged circular pipe. *Physical Review*, 73(4):383–406, 1948.

[3] M. L. Munjal. *Acoustics of Ducts and Mufflers.* John Wiley & Sons, New York, 1987.

[4] O. Zienkiewicz and R. Taylor. *The Finite Element Method.* Butterworth-Heinemann, 2002.