Hysteresis and Creep Compensation For Piezoelectric Actuators Applied to the Feedforward Control Command of Flexible Structures

J. Becker, T. Krämer, L. Gaul
Institut für Angewandte und Experimentelle Mechanik, Universität Stuttgart, Email: {becker,gaul}@iam.uni-stuttgart.de

Introduction
Position control of flexible structures is of major interest for many applications, e.g. adaptive optics, nanomotion with flexures, high-speed valves or MEMS. Typically for them is the demand for high dynamics and high efficiency which has made actuation based on piezoelectric materials popular in the last years. Unfortunately, these materials exhibit hysteresis and creep effect if they operated with electric fields outside the small-signal regime significantly degrading control performance.

Driven from this observation, a feedforward control design methodology with special emphasis on the appropriate compensation of the large–signal piezoelectric effects, i.e. hysteresis and creep, is proposed in this contribution. It is verified in simulations and experiments for a test structure: a cantilever beam actuated by a bonded piezoelectric patch as shown in Fig. 3. The patch is operated in the 31-mode in order to bend the structure, i.e. the electric field is applied in out–of–plane direction and leads to in–plane strains. As a typical positioning control task, fast transition of the beam tip from initial stationary zero deflection to a prescribed stationary deflection is considered. For linear piezoelectricity, previous work can be found in [1, 2] and similar results, although restricted analytically tractable models, in [3].

Feedforward Control Design
The design of the feedforward control command is conducted in two steps exploiting the fact that the piezoelectric structure as control plant can be separated into a series connection of a input nonlinearity and a linear system part as shown in Fig. 2.

In the first step, the nonlinear piezoelectric effects are modelled. Based on the identified models, a discrete–time inverse filter is designed in order to compensate the nonlinear hysteresis and creep effects. This filter is inserted at the input of the piezoelectric structure which yields a linearized overall system (hereby, linearization does not mean Taylor expansion around a working point).

In the second step, the feedforward control is designed based on this linearized system dynamics, which is obtained here by appropriate finite-element discretization of the piezoelectric structure (shell elements for beam, bulk elements for piezoelectric material, see Fig. 3). Based on modal representation of the dynamics and a flatness-based control design, a feedforward tracking control \( u(t) \) is designed that makes the beam tip \( w(t) \) follow a prescribed trajectory \( w^*(t) \) which is obtained by appropriate motion planning for the flat output \( y^*(t) \) and use of the flatness-based parameterizations. Excellent tracking performance is observed if the control command is applied to the full–order linear model as already shown in [2]. However, a minimal transition time depending on beam length and bending wave speed must be respected in the feedforward control design procedure.

The theoretical background of the feedforward control design, especially the connection between analytical partial differential equations describing the dynamics and the flatness–based feedforward tracking control design, is studied in [1, 2]. Shortly described, a virtual flat output \( y(t) \) is determined, which is then used to parameterize the system states and system input \( u(t) \) by the flat output \( y^*(t) \) and its time derivatives.

Motion planning is performed for this flat output, i.e. a sufficiently smooth desired trajectory \( y^*(t) \) is assigned in order to perform rest–to–rest transition. After the flat output \( y^*(t) \) and its time derivatives are determined, the feedforward control \( u^*(t) \) can be calculated by purely algebraic equations as well as the desired system state tra-
jectories and the physical output \( w^*(t) \) are obtained. Finally, the feedforward tracking control \( u^*_c(t) \), i.e. the applied voltage to the piezoelectric patch, is obtained by the filtering of the flatness–based feedforward control \( u^*(t) \) with the inverse filter \( P^{-1} \) (see Fig. 2).

### Hysteresis and Creep Modelling and Compensation Filter Design

The nonlinearities \( P \) can be modelled by superposition of a hysteresis operator \( H[\cdot] \) and a creep operator \( K[\cdot] \), see e.g. [4], according to

\[
    u_c(t) = P[u(t)] = H[u(t)] + K[u(t)].
\]

For modelling the static hysteresis \( H \), inflection points marking the boundary of the hysteresis loops are updated by detecting direction reversals of the input signal. An outer hysteresis loop (consisting of a loading and unloading branch) is fitted by 3rd order polynomials. Then, the inner hysteresis loops are described by scaling of these polynomials with regard to the inner inflection points (ip). These points are updated by detecting direction reversals of the input signal and implementation of the Madelung rules for rate-independent hysteresis according to the scheme in Fig. 4.

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**Figure 4:** Scheme of hysteresis loop with reversal points as used by the implementation of the hysteresis model. The inverse operator \( H^{-1} \) is similarly implemented by mirroring the hysteresis branches and reversal points at the angle bisector. In combination with the creep compensation, which is performed by application of a 4th–order Kelvin-Voigt model, the compensation filter \( P^{-1} \) (inverse filter) can be written as

\[
    u_c(t_k) = P^{-1}[u^*(t_k)] = H^{-1}[u^*(t_k)] - K[u_c(t_{k-1})].
\]

For slowly varying input voltages, the beam dynamics can be considered to be a static gain \( r = w(\infty)/u(\infty) \). In Fig. 5, the achieved compensation is investigated for such inputs of different amplitudes. As expected, the hysteresis ”waist” is reduced by a factor 5-10.

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**Figure 5:** Verification of creep and hysteresis compensation: Measured tip deflections and hysteresis loops for slowly varying input signals as used in the identification.

### Feedforward Tracking Control Results

The flatness–based feedforward control is applied to the beam structure according to Fig. 2. A transition in \( T = 20 \) ms is investigated which is very fast compared to the beam dynamics with the first bending mode eigenfrequency located at approx. 25 Hz.

In Fig. 6, the measured deflection time signals with and without compensation of the creep and hysteresis effects are shown. Obviously, the tracking performance is greatly improved over the investigated range of step heights (corresponding to different voltages ranges), i.e. the nonlinear filter strongly reduces residual vibrations excited during the transition interval as well as the stationary offset observed at the final state.

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**Figure 6:** Comparison of the applied feedforward control signals and the corresponding obtained measured deflections.

### Conclusions

The combination of a nonlinear inverse filter for hysteresis and creep compensation with a flatness–based feedforward tracking control design is proposed for positioning tasks using piezoelectric flexures. This approach is experimentally verified for the rest-to-rest positioning of a beam tip by means of an attached piezoelectric patch actuator. The proposed inverse filtering approach successfully compensates the nonlinear piezoelectric effects, by which it improves the flatness–based feedforward control for operation outside the small–signal. The presented methodology may be applied to a large variety of piezoelectric structures and applications.

### References


