Energy Finite Element Methods for Room Acoustics

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Introduction

The application of numerical methods like FEM or BEM to room acoustical or building acoustical problems is (at least for the majority of rooms larger than e.g. vehicle enclosures) up to now limited to a frequency range that is considerably smaller than the range of commonly used room acoustical analyses.

The limiting factor for these methods is the required discretization of the volume or surface in elements of a size small enough to reconstruct the wave in magnitude and phase at every point of the discretization domain. An usual rule of thumb requires a discretization of at least six nodes per wavelength in each spatial direction. This limitation may be lifted by using methods which do not model the phase of the sound field in the room, but only its energy or intensity distribution. The spatial variation of these quantities is much lower than the variation of the phase, so coarser discretizations are sufficient. One typical method of this type is the Energy Finite Element Method [1, 2].

This paper shows a first, very simple application of an Energy Finite Element Method in room acoustics, with some comparisons to measured and theoretical results.

Theoretical Background

The Energy Finite Element Method (EFEM) as introduced in [2] assumes broadband diffuse sound fields composed of random incidence plane waves. This is an acceptable approximation for rooms with low absorption, broad spatial distribution of absorption and for locations far enough away from sound sources or reflecting/absorbing wall structures.

The fundamental equation for EFEM (and most other energy based methods) is the equilibrium of energy in a differential subsystem:

$$P_{\text{in}} = \nabla\vec{I} + P_{\text{diss}} \tag{1}$$

where P_{in} is the input power into the subsystem, $\vec{I}$ is the vector of intensity, and P_{diss} is the internally dissipated power in the subsystem.

To relate energy density to intensity, some small amount of internal dissipation must be included. The damping model employed is the model of hysteretic damping, relating the time averaged power dissipation $\overline{P_{\text{diss}}}$ to the time averaged energy density $\overline{w}$ by

$$\overline{P_{\text{diss}}} = \omega\eta\overline{w}\,. \tag{2}$$

The relation between sound intensity and energy density in a diffuse field, assuming some internal dissipation, is

$$\vec{I} = -\frac{c^2}{\eta\omega}\nabla w\,. \tag{3}$$

Combining the previous equations results in the following parabolic PDE, which is quite similar to the heat transfer or diffusion equation:

$$-\frac{c^2}{\eta\omega}\Delta w + \eta\omega w = P_{\text{in}} \tag{4}$$

EFEM element construction

The parabolic PDE can be discretized and transferred easily into a FEM system. Defining the EFEM system as

$$\left(\mathbf{K} + \omega\mathbf{C} + \omega^2\mathbf{M}\right)\mathbf{w} = \omega\left(\mathbf{p} - \mathbf{b}\right) \tag{5}$$

with the system matrices $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$, the vector $\mathbf{p}$ as a vector of power sources and $\mathbf{b}$ as a boundary intensity vector, the element matrices can be calculated as follows:

$$K_{ij} = \int_V \frac{c^2}{\eta}\nabla\Phi_i\nabla\Phi_j dV \tag{6}$$

$$M_{ij} = \int_V \eta\Phi_i\Phi_j dV \tag{7}$$

The flow of energy into absorbing walls can be modeled approximately by employing the Sabine model. The intensity of a wave absorbed by a wall in a diffuse sound field is related to the diffuse energy density by

$$\vec{I}\vec{n} = \frac{\alpha}{4}c\,w\,. \tag{8}$$

This relation can be implemented in the damping matrix $\mathbf{C}$ by evaluating

$$C_{ij} = \int_{\partial V}\frac{\alpha}{4}c\Phi_i\Phi_j dS\,. \tag{9}$$

Finally, an intensity source with a normal intensity I_{B} can be transferred into its boundary source vector $\mathbf{b}$ by

$$b_i = -\int_{\partial V} I_{\text{B}}\Phi_i dS\,. \tag{10}$$

Examples

To examine the performance of the EFEM method, two example problems have been calculated. The first example is a shoebox shaped room with dimensions $7 \times 5 \times 3$ m, discretized with parabolic tetrahedron elements, with 1050 nodes. A point power source in one corner of the room was used as the input, and varying values of absorption were applied on all six walls of the room.

According to classical theory, the steady state diffuse field energy in such a room is $w_D = 4P/(cA)$, where P is the inserted sound power, and A is the equivalent absorption area. For some different values of wall absorption and an input power

Table 1: Comparison: EFEM vs. theoretical results, shoebox room

α	$w = 4P/(cA)$ [J/m^3]	w (EFEM) [J/m^3]
0.1	$8.196 \cdot 10^{-4}$	$8.172 \cdot 10^{-4}$
0.3	$2.732 \cdot 10^{-4}$	$2.729 \cdot 10^{-4}$
0.5	$1.639 \cdot 10^{-4}$	$1.638 \cdot 10^{-4}$
0.9	$9.107 \cdot 10^{-5}$	$9.103 \cdot 10^{-5}$

$P = 1$ W, the simulation results at $f = 100$ Hz are shown in Table 1. The EFEM results for w have been obtained at a node near the center of the room.

A more detailed examination of the results shows very low spatial or frequency variation of energy density. This can be explained by the low dissipation losses in the room volume.

As a second model, a very detailed model of a recording studio (mermaid music studio, Munich), which has also been used in low- to mid-frequency FEM calculations in previous research, was used as a practical example to check the performance and accuracy of EFEM the method. This model, shown in Figure 1, has a detailed geometry, and frequency varying absorption data on most surfaces.

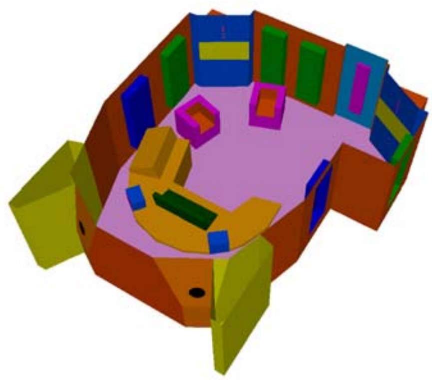

Figure 1: CAD Model of mermaid music studio, Munich

The comparison at the typical listener position in Figure 2 shows quite a good agreement in a broad frequency range, the systematic deviation in the frequency range above 800 Hz can be explained by an incomplete measurement of loudspeaker power in this frequency range. The comparison near an absorbing surface shown in Figure 3, reveals larger differences to the measured results. Here, the diffuse field assumption is not valid. Again, there is no noticeable spatial variation of sound energy in the EFEM results. The visible frequency response is mostly determined by the variations of absorption and input power.

Conclusions

The EFEM, as shown in this paper, is able to calculate correctly the diffuse field of large rooms very quickly. The calculation of diffuse field energy is in very good agreement with classical theory. The method can be used on any existing FEM mesh. As other works have shown, the EFEM is also an efficient method to calculate energetic sound transmission through large and complex structures, and the acoustical EFEM can be easily coupled with the structural EFEM to determine sound energy levels in cavities inside the structure.

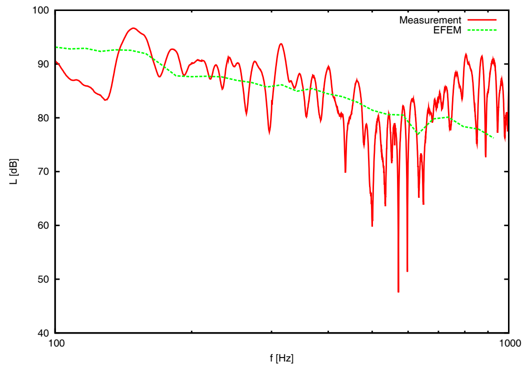

Figure 2: Comparison at listener position

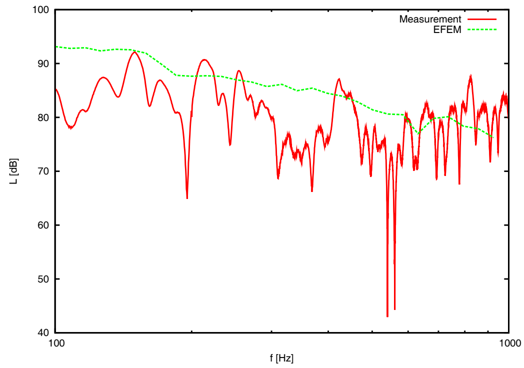

Figure 3: Comparison at position near absorbing surface

The problems of the acoustical EFEM inflicted by its strict plane wave diffuse field assumption. The method is not able to show much spatial variation of sound energy in the field, at least for moderately sized rooms and lower frequencies. The source directivity cannot be easily modeled in the context of EFEM, and the direct energy in the near field of the source is not considered.

Outlook

Some approaches could be used to overcome the limitations of EFEM. The EBEM method [3] does take spreading waves from the source and from specular reflections into account, thereby giving results which include some amount of near field effects. Another approach could be the application of geometrical models (ray tracing, image source models) after an automatic model simplification. Finally, wave element based methods could enable a correct amplitude and phase solution of mid- to high wave number problems.

References

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[3] Franzoni, L. P., Bliss, D. B. and Rouse, J. W..: An acoustic boundary element method based on energy and intensity variables for prediction of high-frequency broadband sound fields. J. Acoust. Soc. Am. **110** (6), December 2001.