The Space Time Expansion discontinuous Galerkin method

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Our research objectives are the construction and application of accurate and efficient schemes for nonlinear wave propagation. We presented the recently, see [1], [2], developed space-time expansion discontinuous Galerkin (STE-DG) approach for the unsteady compressible Navier-Stokes equations. The discontinuous Galerkin methods are locally conservative, L_2-stable and high-order accurate methods, which can easily handle complex geometries, meshes with hanging nodes and elementwise different polynomial approximations. These properties render them ideal to be used with *hp*-adaptation and, as independent of the polynomial approximation degree only data from direct neighbours are needed, well suited for massive parallel computations. The basis of our scheme is a spatial discontinuous Galerkin formulation, where special care for the second order terms is taken. Using integration by parts two times, adjoint consistency of the discretization is achieved without introducing auxiliary variables. The spatial polynomial of the DG approach is expanded in time using the so called Cauchy-Kovalevskaya (CK) procedure. With a polynomial of order N in space the CK procedure generates an approximation of order N in time as well yielding a scheme being accurate of order $N+1$ in space *and* time. The surface and volume integrals in the variational formulation are approximated by Gaussian quadrature with the values of the space-time approximate solution. The numerical fluxes at grid cell interfaces are based on the approximate solution of generalized Riemann problems for both, the inviscid and viscous part. The fully explicit scheme has to satisfy a stability restriction similar to all other explicit DG schemes. The loss of efficiency, especially in the case of strongly varying sizes of grid cells is circumvented by use of different time steps in different grid cells. Due to the locality and the space-time nature of the presented method it is possible to introduce time accurate local time stepping. We drop the common global time step and propose for a time-dependent problem that any grid cell runs with its own time step determined by the local stability restriction. In spite of the local time steps the scheme is fully explicit, conservative, and as in the DG approach the polynomial order could be chosen arbitrarily, the scheme is theoretically of arbitrary order of accuracy in space and time for transient calculations. The fields of applications are computational fluid dynamics (CFD) in combination with computational aeroacoustics (CAA). Due to the local time stepping feature, the scheme enables us to efficiently perform direct simulations of aeroacoustic problems, such as the sound generation of a cylinder van Karman vortex street. In figure 1 the acoustic pressure and velocity magnitude of such an example is shown, respectively. For this simulation we choose the mach number $M = 0.2$ and the free stream Reynolds number based on the diameter $D = 1$ of the cylinder $Re_D = 150$. In

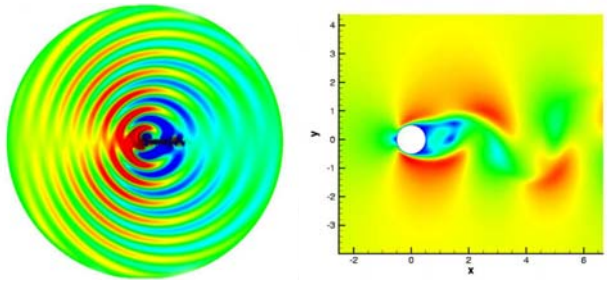

Figure 1: Acoustic pressure and velocity magnitude.

figure 2 the *hp*-adapted grid and distribution of the polynomial approximation degree is shown. Red indicates an approximation of 5th order polynomials and blue an approximation with 2nd order polynomials. To capture the farfield acoustic, large cells with high order approximations are chosen. We found out, that due to the local time stepping the acoustic farfield part of the grid needs less than 0.5 percent of the overall CPU time, confirming that the STE-DG scheme is well suited for direct simulation of aeroacustic.

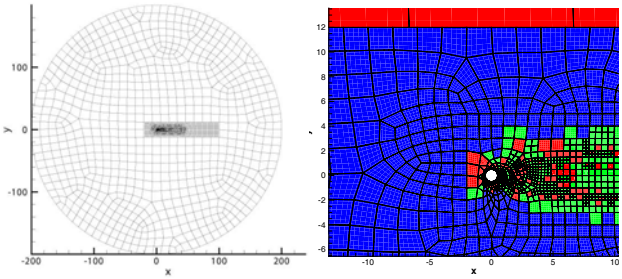

Figure 2: Computational grid.

References

[1] F. Lörcher, G. Gassner and C.-D. Munz, *A discontinuous Galerkin scheme based on a space-time expansion. I. Inviscid compressible flow in one space dimension*, J. of Sci. Comp., doi:0.1007/s10915-007-9128-x.

[2] G. Gassner, F. Lörcher and C.-D. Munz, *A discontinuous Galerkin scheme based on a space-time expansion. II. Viscous flow equaqtions in multi dimensions*, submitted to J. of Sci. Comp.