Can also sound be handled as stream of particles? – An improved energetic approach to diffraction based on the uncertainty principle - from ray to beam tracing

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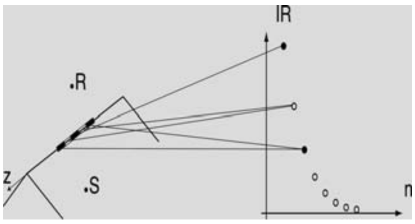
Introduction

In room acoustics as well as in noise immission prognosis (‘city acoustics’) ray or beam tracing, or hybrid methods, are in use, methods made for the optical limiting case of ‘small’ wavelengths. This shall be assumed here as well, extended to larger wavelengths, but also incoherence and energetic superposition. The main deficiency is then the lack of simulation of diffraction. Wanted is the introduction of an edge diffraction module, at least as an approximation, fulfilling the ‘detour law’ [1], but also for arbitrary orders and combinations with reflections, where the detour law fails. But usually, any combination with the mirror image source method (MISM) [2] leads to an explosion of the number of rays and computation time, also a combination with the sound particle method [3]. Beam tracing, as an efficient version of the MISM, offers a better chance for that [4]. Therefore, the Uniform Theory of Diffraction, a high frequency approximation, has already been utilized [5].

The coherent secondary edge source model

Svensson [6] succeeded to derive analytical directivity functions (β) from an exact time-domain solution for an infinite rigid wedge based on an approach of [7], extended by the idea of secondary edge sources [8]. The discretized impulse response (for samples no. n) can be written as

$$IR(n) = -\frac{1}{4\Theta_w}\int_{z_1}^{z_2}\frac{\beta}{ml}\,dz$$



where θ_W is the exterior cylindrical angle of the wedge, m and l are distances to and from the edge source, z_1 and z_2 the integration range limits on the edge (=z-axis), which also may be finite, see Fig. 1. This is even valid for lower frequencies. It can be recursively applied for higher order – but with inaccuracies. The problem of the computation time explosion is not solved either.

The Sound Particle-Edge Interaction Model

One of the basic ideas to solve that problem is: not all combinations and paths of diffracted/reflected rays are important, only those where ray pass edges near by. So, an efficient straight forward method as ray tracing seems suitable. This is the idea of an ‘edge interaction model’ (SPEIM), Stephenson evaluated already in 1986 [9], , however only for - receivers at infinite distance.

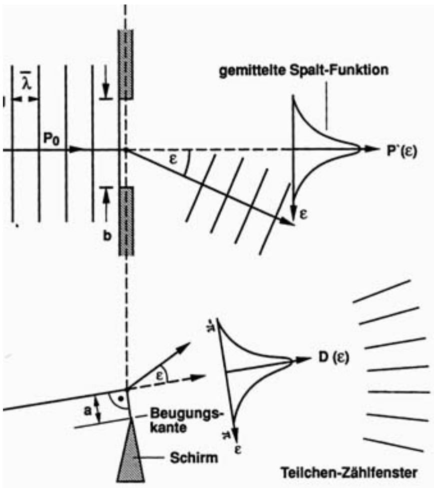
Quantized Pyramidal Beam Tracing (QPBT)

But the crucial problem of recursive split-up of rays with each diffraction event and hence computation time explosion had not been solved. In 1996, Stephenson invented QPBT - a method to efficiently re-unify partially overlapping pyramidal beams (by quantizing space and solid angles in a kind of iterative radiosity algorithm) [10]. But this does not work with thin rays, spatially extended beams are required. Therefore there was now the need for a universal beam diffraction method based on edge interaction. To perform this efficiently, a subdivision of the room into convex sub-rooms is proposed where on the ‘transparent’ dividing walls diffraction events at ‘inner edges’ may be detected. So the SPEIM was re-evaluated, for the first time embedded in a full ray tracing program - now also for more general set-ups –later extended to a more efficient beam tracing model.

The application of the uncertainty principle

The bending effect on a sound particle is stronger the nearer the by-pass-distance to the edge. This idea is inspired by Heisenbergs Uncertainty-Relation UR, known from quantum mechanics: $\Delta x \cdot \Delta p_x \approx h$ where Δx is the by-pass distance, interpreted as the ‘uncertainty’ in x, Δp_x the impulse uncertainty in x and h Planck`s constant/2π . One may object: acoustics is not happening in atomic scales. But dividing the UR through h (using de Broglie’s equation $\Delta p_x = h \cdot \Delta k_x$) yields $\Delta x \cdot \Delta k_x \approx 1$ -without any atomic constant - a consequence also from the Fourier theorem. $\Delta k_x / k$ is then the uncertainty of the direction of the wave vector – valid also for acoustics! The UR can be utilized to create diffraction algorithms for any kind of particles, phonons as well as photons. This idea has already been successfully utilized in numerical methods for light diffraction to optimize optical systems [11 based on 12]. The idea to implant that into a ray model is to make a ‘detour into wave theory’:

Fig.2: The sound particle diffraction model (Stephenson 1986): In the moment a particle passes an edge with distance a (see below), it ‘sees’ a slit (see above) of relative width b=6a (all distances in the following are relative compared with wavelength) (‘6’ follows from a self-consistency-consideration).

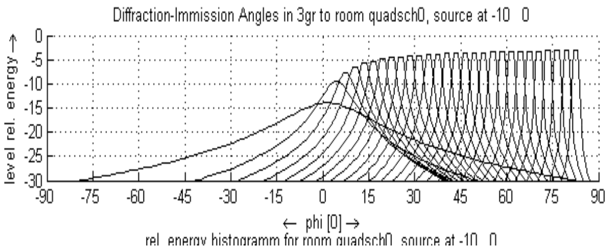


A slit diffraction function is applied.

This is derived from the Fraunhofer diffraction at a slit $\propto \sin^2 \nu / \nu^2$, octave band averaged (for "white light"). The result is simply $D(\nu) = D_0 / (1 + 2\nu^2)$ with $\nu = 2 \cdot b \cdot \varepsilon$, $\boldsymbol{\varepsilon}$= deflection angle, D_0 =normalization factor (the integral over all angles is 1, see curve in fig. 2 above). This may be used as a 'Diffraction angle probability density function (DAPDF) or, more efficiently, to define the energies of secondary particles. To develop a modular model, applicable also to several edges passed near by 'simultaneously', an 'Edge Diffraction strength' (EDS) $EDS(a) = 1/(6 \cdot a)$ is introduced such that the EDS of several edges may be added up to a total TEDS and an 'effective slit width' is $b_{eff} = 1/TEDS$.

Results of generalized ray diffraction experiments

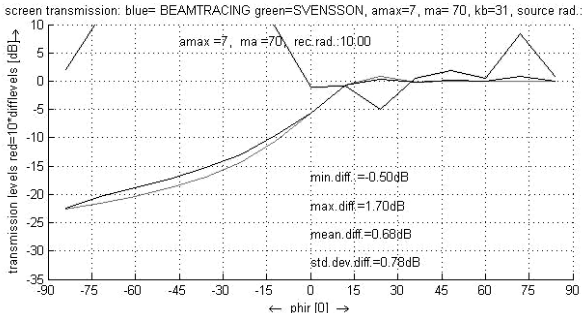
This particle diffraction model has now been combined with a full 2D sound particle tracing algorithm [3], with sources and receivers also at finite distances of 1,3,10,30,100 (wavelengths) and 15 angles from -90…90°, applied to the semi-infinite screen (1 edge) and compared with the known angle function of the screen [1]. Fig. 3 shows the superposition of



the DAPDFs of the single particles passing by in different distances (see fig. 2) summing up to the screen function (as shown in fig. 4). At the first go the agreement with the reference function was very good almost in all cases (standard deviation <0.5dB). It can be proved also that the reciprocity principle is fulfilled. Numerically decisive is the number of incident particles within close by-pass distances, a_{min} should be about 0.1λ. What is very handy: A maximum by-pass distance of $a_{max} = 7\ \lambda$ is enough, beyond which direct transmission may be performed. The orientation of the transmitting (deflecting) surface has only a weak influence.

From ray to beam tracing

Ray (or rather particle) tracing requires spatially extended detectors as receivers and, to reach a certain numerical accuracy, a higher number of particles crossing each detector (about 70 for 0.5dB). More decisive: only spatially extended beams may overlap and give the chance for the urgently required re-unification effect of QPBT. Since beams are 'mirror image sources with built-in visibility limits' just the $1/r^2$ -distance law may be applied to compute the immitted intensities at the receiver points (in 2D a 1/r-law). Therefore, only one loop over all beams incident near the edge is necessary, not a secondary loop over each time an additional number of secondary particles. So, a 2D beam tracing algorithm was implemented, specialized for the set-up of one or two edges (a slit). The incident beams - in 2D rather 'fans' - were, distance-dependent, narrow enough such that typically 10…100 arrived in the decisive by pass-distance range of $0 \ldots 7\ \lambda$. Each of them was split up into 15 secondary beams (preferrably the same number as receiver directions on the other side). A criterion for the valid by-pass distance is, as a compromise, the middle ray's distance within the beam. For the above mentioned 5*5*15 source- receiver position combinations, comparisons were carried out with the former ray tracing method, with the Maekawa screen transmission function, the slit-function itself (as a self-consistency test), and, finally, with Svensson's exact analytical results for the (hard) wedge. The agreements were in all cases very good. Fig. 4 shows the (receiver-) angle-dependent transmission values (in dB) for the case of source and receiver distances of 10λ. (To the left in the 'shadow' region, the two almost coinciding curves are



for beam tracing and Svensson's method, the third curve shoes their difference exaggerated by the factor 10). The standard deviation for all combinations was **only 0.39dB!**

Conclusions and outlook

Heisenberg's UR may obviously be applied also to acoustics. It seems sound may be handled as particles even with diffraction. It should not be a principal problem to extend the presented model to 3D and to multiple diffractions. (According to the UR, edge diffraction happens only in the area perpendicular to the edge.) So, a combination with QPBT is now possible. The application to room and city acoustics comes closer.

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