An Analytic Secondary Source Selection Criterion for Wavefield Synthesis

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Introduction

Wavefield synthesis (WFS) is a spatial sound reproduction technique that facilitates a high number of loudspeakers (secondary sources) to create a virtual auditory scene for a large listening area. It requires a sensible selection of the active secondary sources used for the reproduction of a particular virtual source. For virtual point sources and plane waves suitable intuitively derived selection criteria are used in practical implementations. However, for more complex virtual source models, like presented e. g. in [1], the selection might no be straightforward.

In previous publications of the author [2, 3] a secondary source selection criterion was formulated non-analytically. This contribution proposes to use the sound intensity vector of the virtual source wavefield in order to formulate an analytic secondary source selection criterion. It is shown that the selection schemes currently used in practical implementations are verified by the proposed criterion.

Wavefield Synthesis

The physical basis of sound reproduction is given by the Kirchhoff-Helmholtz integral [4]

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \left(G_0(\mathbf{x}|\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega) - S(\mathbf{x}_0,\omega)\,\frac{\partial}{\partial \mathbf{n}} G_0(\mathbf{x}|\mathbf{x}_0,\omega) \right) dS_0 \,, \qquad (1)$$

where $P(\mathbf{x},\omega)$ denotes the pressure field inside the bounded region V surrounded by the border ∂V, $G_0(\mathbf{x}|\mathbf{x}_0,\omega)$ the free-field Green's function, $S(\mathbf{x},\omega)$ the wavefield of the virtual source and $\frac{\partial}{\partial \mathbf{n}}$ the directional gradient in direction of the normal vector $\mathbf{n}$. Figure 1 illustrates the parameters. The Green's function $G_0(\mathbf{x}|\mathbf{x}_0,\omega)$ characterizes the wavefield emitted by a point source (monopole) placed at the position $\mathbf{x}_0$, its directional gradient as the field of a dipole point source. The Kirchhoff-Helmholtz integral can be interpreted as follows: A distribution of monopole and dipole sources placed around a desired listening area V is sufficient for recreation of a desired virtual source within the entire listening area.

The second term in the Kirchhoff-Helmholtz integral (1) belonging to the dipole secondary sources can be eliminated by modifying the Green's function used [2, 3]. The modified Green's function $G'(\mathbf{x}|\mathbf{x}_0,\omega)$ has to obey the following condition

$$\frac{\partial}{\partial \mathbf{n}} G'(\mathbf{x}|\mathbf{x}_0,\omega)\Big|_{\mathbf{x}_0 \in \partial V} = 0 \,. \qquad (2)$$

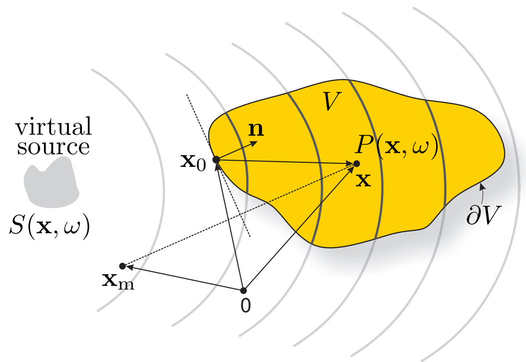

Figure 1: Geometrical parameters used for the Kirchhoff-Helmholtz integral (1) and monopole only reproduction.

Condition (2) formulates a homogeneous Neumann boundary condition imposed on ∂V, hence the boundary ∂V will be implicitly modeled as acoustically rigid surface. The desired Green's function can be derived by adding a suitable homogeneous solution (with respect to the region V) to the free-field Green's function

$$G'(\mathbf{x}|\mathbf{x}_0,\omega) = G_0(\mathbf{x}|\mathbf{x}_0,\omega) + G_0(\mathbf{x}_\mathrm{m}(\mathbf{x})|\mathbf{x}_0,\omega) \,. \qquad (3)$$

A solution fulfilling Eq. (2) is given by choosing the receiver point $\mathbf{x}_\mathrm{m}(\mathbf{x})$ as the point $\mathbf{x}$ mirrored at the tangent to the curve ∂V at the position $\mathbf{x}_0$ [2, 3]. Please note that due to the specialized geometry $G'(\mathbf{x}|\mathbf{x}_0,\omega) = 2\,G_0(\mathbf{x}|\mathbf{x}_0,\omega)$.

As mentioned above, the surface ∂V will be implicitly modeled as rigid surface. Hence, the reproduction of the desired wavefield will additionally reproduce undesired reflections at the (virtual) boundary ∂V. These reflections will only take place for those components where the local propagation direction of the wavefield to be reproduced does not coincide with the normal vector $\mathbf{n}$. Since we are free to choose the secondary sources used for reproduction, these undesired reflections can be avoided by discarding those secondary sources which reproduce the reflections. This selection can be formulated by introducing the window function $a(\mathbf{x}_0)$ into the secondary source driving function $D(\mathbf{x}_0,\omega)$

$$P(\mathbf{x},\omega) = -\oint_{\partial V} \underbrace{2a(\mathbf{x_0})\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0,\omega)}_{D(\mathbf{x}_0,\omega)}\; G(\mathbf{x}|\mathbf{x}_0,\omega)\; dS_0 \,, \qquad (4)$$

where $a(\mathbf{x}_0) = 1$ if $\mathbf{n}$ coincides with the local propagation direction of the wavefield $S(\mathbf{x}_0,\omega)$ at the position $\mathbf{x_0}$, and $a(\mathbf{x}_0) = 0$ otherwise.

This definition of the window function is obvious but not

useful for practical reproduction algorithms since it is not formulated analytically. The following section will introduce an analytical definition of the window function $a(\mathbf{x}_0)$.
The theory as presented so far is not limited to WFS, it also holds for other sound reproduction systems. Two-dimensional WFS utilizes approximations of secondary line sources by point sources (closed loudspeakers) for reproduction. Please see e. g. [2, 3] for more details. However, these approximation will have no influence on the proposed selection scheme.

Secondary Source Selection Criterion

The basic idea to derive an analytic secondary source selection criterion is to utilize the acoustic intensity vector $\mathbf{i}(\mathbf{x}, t) = p(\mathbf{x}, t)\,\mathbf{v}(\mathbf{x}, t)$ for selection of the active secondary sources. The acoustic intensity vector $\mathbf{i}(\mathbf{x}, t)$ represents the instantaneous amount of energy flow per unit time and area with respect to direction. The intensity vector points into the direction of increased energy density. Hence, $\mathbf{i}(\mathbf{x}, t)$ can be used to characterize the traveling direction of acoustic waves. However, it is useful to average the intensity vector $\mathbf{i}(\mathbf{x}, t)$ over a signal period. Time averaging is conveniently formulated in the frequency domain. The time averaged acoustic intensity vector $\bar{\mathbf{I}}_S(\mathbf{x}, \omega)$ for the virtual source is given as [4]

$$\bar{\mathbf{I}}_S(\mathbf{x}, \omega) = \frac{1}{2}\Re\{S(\mathbf{x}, \omega)\mathbf{V}_S(\mathbf{x}, \omega)^*\} \,, \qquad (5)$$

where $\Re\{\cdot\}$ denotes the real part of the argument and the superscript $*$ the conjugate complex function. The window function is defined straightforward using the time-averaged acoustic intensity as

$$a(\mathbf{x}_0) = \begin{cases} 1 & \text{, if } \langle \bar{\mathbf{I}}(\mathbf{x}_0, \omega), \mathbf{n}(\mathbf{x}_0) \rangle > 0, \\ 0 & \text{, otherwise.} \end{cases} \qquad (6)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product of two vectors. In the next section this definition will be applied to two analytic virtual source models frequently used in sound reproduction.

Application to Virtual Source Models

Typical analytic models that are used for virtual sources are plane waves and point sources. The wavefield of a monochromatic plane wave is given as follows [4]

$$S_{\text{pw}}(\mathbf{x}_0, \omega) = \hat{S}(\omega)\; e^{-j\mathbf{k}_{\text{pw}}^T \mathbf{x}_0} \,, \qquad (7)$$

where $\hat{S}(\omega)$ denotes the spectrum of the plane wave, $\mathbf{k}_{\text{pw}} = \frac{\omega}{c}\,\mathbf{n}_{\text{pw}}$ the wave vector of the plane wave and $\mathbf{n}_{\text{pw}}$ its normal vector (propagation direction). Figure 2 illustrates the geometry. The time averaged acoustic intensity for a plane wave is given by introducing (7) and its velocity into (5) as

$$\bar{\mathbf{I}}_{\text{pw}}(\mathbf{x}, \omega) = \left|\hat{S}(\omega)\right|^2 \frac{1}{2\rho_0 c}\,\mathbf{n}_{\text{pw}} \,. \qquad (8)$$

Hence, a secondary source is selected if the normal vector $\mathbf{n}$ of the secondary source and the propagation direction of the plane wave $\mathbf{n}_{\text{pw}}$ form an acute-angle. This is conjunction with the state of the art used in practical implementations.

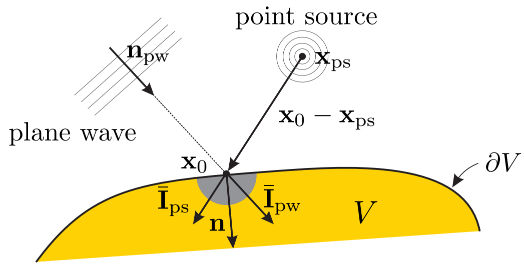

Figure 2: Application of proposed secondary source selection criterion to virtual plane waves and point sources.

The wavefield of a point source is given as follows [4]

$$S_{\text{ps}}(\mathbf{x}_0, \omega) = \hat{S}(\omega)\; \frac{1}{r}\, e^{-j\frac{\omega}{c}r} \,, \qquad (9)$$

where $r = |\mathbf{x}_0 - \mathbf{x}_{\text{ps}}|$ denotes the distance between the secondary source position $\mathbf{x}_0$ and the virtual source position $\mathbf{x}_{\text{ps}} \notin V$. The time averaged acoustic intensity for a point source is

$$\bar{\mathbf{I}}_{\text{ps}}(\mathbf{x}, \omega) = \left|\hat{S}(\omega)\right|^2 \frac{1}{2\rho_0 c}\; \frac{1}{r^2}\; \underbrace{\frac{\mathbf{x}_0 - \mathbf{x}_{\text{ps}}}{|\mathbf{x}_0 - \mathbf{x}_{\text{ps}}|}}_{\vec{e}_r} \,, \qquad (10)$$

where $\vec{e}_r$ denotes the outward pointing radial vector of the point source. Hence, a secondary source is selected if the normal vector $\mathbf{n}$ of the secondary source and the outward pointing radial vector $\vec{e}_r$ of the point source form an acute-angle.
The results derived in this section show that the proposed selection criterion (6) is in conjunction with selection schemes used in practical WFS implementations.

Conclusion

This paper presents an analytic secondary source selection criterion for WFS. It has been shown that the proposed criterion verifies the selection schemes applied in current state of the art implementations. The main benefits of an analytic criterion is its applicability to arbitrary virtual source wavefields and simple implementation. Further research in this area includes a generalization to focused point sources ($\mathbf{x}_{\text{ps}} \in V$).

References

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