Reduced-Order Analysis of Turbulent Jet Noise

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Introduction

Targeting noise reduction, the goal of our modelling efforts is to provide a mechanistic understanding of noise generation in turbulent, subsonic jets. Thus, we perform flow decompositions, representing several optimal principles. In particular, a noise optimized flow decomposition into "loud" and "quiet" modes is performed by an extension of the proper orthogonal decomposition (POD) termed the most observable decomposition (MOD).

Data base

The hydrodynamic near-field and the aeroacoustic far-field are provided by numerical simulations. An ensemble of 725 snapshots of the three-dimensional velocity field \( \mathbf{u} \) of a \( Ma=0.9, Re=3600 \) turbulent jet is calculated by a large eddy simulation [1]. The aeroacoustic far-field \( p \) is represented by 76 pressure sensors, situated at a line parallel to the jet axis (see Fig. 1). The sensor signals are determined by a Ffowcs Williams-Hawkins solver. The energy of a Ma=0.9, Re=3600 turbulent jet is calculated by a large eddy simulation [1]. The aeroacoustic far-field is shown. More than 95% of TKE is contained in 21 azimuthal subspaces. The acoustically most important

\[
K_{2D}(x) = \int_{\Omega} dy \, dz \frac{(u' \cdot u')}{2}, \quad Q_{2D}(x) = \int_{\Omega} dy \, dz \frac{(L' \cdot L')}{2}, \tag{2}
\]

and \( Q_{2D}(x) \)-curve is more pronounced and the curve drops off faster towards the end of the domain. This result indicates a bounded spatial extension of the acoustically active region, whereas the kinetic energy \( K_{2D}(x) \) is still high in a downstream region.

Azimuthal Fourier decomposition

An azimuthal Fourier decomposition is performed on the snapshot ensemble. In Fig. 3 the distribution of turbulent kinetic energy \( K_{\Omega} \) (TKE) and aeroacoustic source level \( Q_{\Omega} \) (ASL)

\[
K_{\Omega} = \int_{\Omega} dx \frac{(u' \cdot u')}{2}, \quad Q_{\Omega} = \int_{\Omega} dx \frac{(L' \cdot L')}{2} \tag{3}
\]

is shown. More than 95% of TKE is contained in 21 azimuthal subspaces. The acoustically most important
events are captured by these subspaces [3]. Moreover, more than 90% of ASL is contained in these 21 azimuthal subspaces.

**Reduced-order representations**

Reduced-order representations of velocity, aeroacoustic source and aeroacoustic far-field are performed utilising POD. Fluctuations are approximated by the linear expansion into POD modes

\[ u' \approx \sum_{i=1}^{K} a_i(t) u_i(x), \quad L' \approx \sum_{i=1}^{L} a_i^L(t) L_i(x), \]  \[ p' \approx \sum_{i=1}^{M} a_i^p(t) p_i(x). \]  

The POD modes \( u_i \) decompose the velocity most efficiently for TKE, whereas the POD modes \( L_i \) decompose the aeroacoustic source field most efficiently for ASL. The far-field POD modes \( p_i \) decompose the aeroacoustic source level most efficiently for the level of aeroacoustic far-field fluctuation, which is here termed noise

\[ Z_{\Gamma}^C = \int_{\Gamma} dy \frac{(p' \cdot p')}{2}. \]  

POD is applied to each of the 21 azimuthal subspaces. Thus 725 \( \cdot \) 21 = 15225 POD modes are determined. The residuals of the resulting reduced-order representation of velocity field and of aeroacoustic far-field are shown in Fig. 4. Only 50% of TKE is resolved by the first 350 POD modes.

**Loud and quiet flow modes**

The POD of near-field velocity and far-field aeroacoustic field are utilised to construct loud and quiet flow modes \( u^*_i \), which decomposes the velocity fluctuation

\[ u' \approx \sum_{i=1}^{N} a_i^*(t) u^*_i(x) \]  

most efficiently for correlated far-field noise

\[ Z_{\Gamma}^C = \int_{\Gamma} dy \frac{(p' \cdot p')}{2}, \]  introducing a relationship \( p' \) of near- and far-field fluctuations. Thus, the construction of loud and quiet mode is attributed to the modelling of this relationship. In the MOD, a linear relationship

\[ p' = C u \]  

is proposed. \( C \) is produced by linear stochastic estimation, based on the Fourier coefficients of near- and far-field POD, see [4]. The MOD modes are thus defined to be the pseudoinverse images of far-field POD modes

\[ u^*_i = C^{-1} p_i. \]  

The MOD resolution of correlated noise is shown in Fig. 5. Surprisingly, more than 90% of the correlated noise is resolved by only 24 MOD modes! Thus, the MOD method enables reduced-order modelling for the control of jet noise dynamics.

Further MOD results are detailed in [4].

**References**


