Active Noise Reduction of Vibroacoustic Systems Using Model Based Control

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Introduction

In recent years acoustic noise reduction has become an important concern in many industrial applications. Noise reduction increases the comfort of a system and reduces the noise radiation into the environment. Consequently, the noise level reduction has become a vital development target, since it contributes to product quality and customer satisfaction. The usual way to reduce the sound emission of a structure is the application of additional damping materials. Such damping materials are not very efficient in the lower frequency range and increase considerably the weight of a structure. The application of smart structural concepts is an alternative way to actively reduce the noise emission. A smart structure consists of a passive mechanical structure and integrated active materials, which in connection with a properly designed controller actively reduce unwanted noise emissions.

The paper first briefly presents an overall finite element approach to the modelling of active vibroacoustic systems. Then it is shown how such a model can be used for controller design purposes, with the focus on model based linear quadratic control. The numerical design process is verified by experimental investigations. Therefore an experimental setup consisting of a rectangular clamped plate with a coupled cavity has been developed. The plate is attached with piezoelectric patch actuators and sensors. The experimental setup enables the measurement of the controlled and uncontrolled sound level in the cavity. The accuracy and the performance of the numerical design approach are evaluated by experimental results.

Finite Element Modelling

The derivation of the finite element formulation for an active structural acoustic analysis is based on the finite element discretized linear equations of motion for the piezoelectric structure [1]

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\boldsymbol{\varphi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_u \\ \mathbf{f}_\varphi \end{bmatrix} \tag{1}$$

and the finite element formulation for the acoustic fluid [2]

$$\mathbf{M}_a \ddot{\mathbf{\Phi}} + \mathbf{C}_a \dot{\mathbf{\Phi}} + \mathbf{K}_a \mathbf{\Phi} = \mathbf{f}_a, \tag{2}$$

where $\boldsymbol{\varphi}$ and $\mathbf{\Phi}$ are the nodal values of the electric potentials and the velocity potentials, respectively and $\mathbf{u}$ is the vector with the nodal structural displacements. The velocity potential Φ is related to the fluid particle velocity v and to the sound pressure p by

$$v = -\nabla^T \Phi, \qquad p = \rho \dot{\Phi}, \tag{3}$$

where ρ stands for the fluid density.

A vibroacoustic coupled formulation results from the subsystems (1) and (2) by introducing additional load vectors. The acoustic pressure acting on the piezoelectric structure represents the additional mechanical load vector

$$\mathbf{f}_{uc} = \mathbf{C}_{uc} \dot{\mathbf{\Phi}} \tag{4}$$

and the structural movement acting on the fluid characterizes the additional acoustic load vector

$$\mathbf{f}_{ac} = -\frac{1}{\rho} \mathbf{C}_{uc}^T \dot{\mathbf{u}}. \tag{5}$$

Adding up the additional load vectors (4) and (5) on the left-hand side of the given equation systems (1) and (2), leads to the vibroacoustic formulation

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho \mathbf{M}_a \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{w}} \\ \ddot{\boldsymbol{\varphi}} \\ \ddot{\mathbf{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{uu} & \mathbf{0} & -\mathbf{C}_{uc} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{C}_{uc}^T & \mathbf{0} & -\rho \mathbf{C}_a \end{bmatrix} \begin{bmatrix} \dot{\mathbf{w}} \\ \dot{\boldsymbol{\varphi}} \\ \dot{\mathbf{\Phi}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} & \mathbf{0} \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_{\varphi\varphi} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho \mathbf{K}_a \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \boldsymbol{\varphi} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_u \\ \mathbf{f}_\varphi \\ -\rho \mathbf{f}_a \end{bmatrix}. \tag{6}$$

Due to the use of the velocity potential Φ instead of the sound pressure p to describe the acoustic fluid, symmetric matrices for the coupled structural acoustic formulation are obtained. In this way effective numerical algorithms can be used for solving. In order to realize the active vibroacoustic analyses, the general purpose FEM software package COSAR [3] was used.

Model Truncation

In general, finite element models contain a large number of degrees of freedom. This feature makes such models not suitable for the design of a controller. Modal truncation based on a few preselected uncoupled structural and acoustical eigenmodes is a widely used technique to reduce the degrees of freedom of a finite element discretized vibroacoustic system. Assuming a Rayleigh damping in both subsystems (1) and (2) and introducing modal coordinates

$$\begin{bmatrix} \mathbf{w} \\ \boldsymbol{\varphi} \end{bmatrix} = \mathbf{Q}_{u\varphi}\mathbf{q}_{u\varphi}, \qquad \boldsymbol{\Phi} = \mathbf{Q}_a\mathbf{q}_a, \qquad (7)$$

where the modal matrices $\mathbf{Q}_{u\varphi} = [\hat{\mathbf{q}}_{u\varphi 1} \quad \hat{\mathbf{q}}_{u\varphi 2} \quad .. \quad \hat{\mathbf{q}}_{u\varphi n}]$ and $\mathbf{Q}_a = [\hat{\mathbf{q}}_{a1} \quad \hat{\mathbf{q}}_{a2} \quad \cdots \quad \hat{\mathbf{q}}_{ak}]$ contain the selected eigenmodes, the truncated system of equations can be written as

$$\mathbf{I}\begin{bmatrix} \ddot{\mathbf{q}}_{u\varphi} \\ \ddot{\mathbf{q}}_a \end{bmatrix} + \boldsymbol{\Delta}\begin{bmatrix} \dot{\mathbf{q}}_{u\varphi} \\ \dot{\mathbf{q}}_a \end{bmatrix} + \boldsymbol{\Lambda}\begin{bmatrix} \dot{\mathbf{q}}_{u\varphi} \\ \dot{\mathbf{q}}_a \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{u\varphi}^T\mathbf{f}_{u\varphi} \\ -\rho\mathbf{Q}_a^T\mathbf{f}_a \end{bmatrix}. \qquad (8)$$

State Space Model and Controller Design

The idea behind modelling the interaction of the fluid with the piezoelectric structure using finite elements is to obtain an accurate model for controller design. To apply a model based control it is necessary to rewrite the reduced formulation (8) in the state space form. Therefore the state space vector $\mathbf{z} = [\mathbf{q}_{u\varphi} \quad \mathbf{q}_a \quad \dot{\mathbf{q}}_{u\varphi} \quad \dot{\mathbf{q}}_a]^T$ is introduced and the system of equations (8) is extended by the matrices $\mathbf{E}_{u\varphi}$, $\mathbf{E}_a$ and $\mathbf{B}_{u\varphi}$, which describe the positions of the external forces $\mathbf{f}$ and the controller forces $\mathbf{u}$, respectively. The state space form reads

$$\dot{\mathbf{z}} = \mathbf{Az} + \mathbf{Bu} + \mathbf{Ef}. \qquad (9)$$

The corresponding measurement equation is given by

$$\mathbf{y} = \mathbf{Cz} + \mathbf{Du} + \mathbf{Ff}. \qquad (10)$$

To complete the control system it is essential to design a controller. A variety of techniques are available to design state feedback control. In this paper a linear quadratic regulator (LQR) is used to determine the controller gains. The goal of the LQR controller is to find a state feedback control law $\mathbf{u}(t)$ that minimizes the cost function

$$J = \int_0^\infty [\mathbf{z}^T(t)\mathbf{Qz}(t) + \mathbf{u}^T(t)\mathbf{Ru}(t)]dt, \qquad (11)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices.

For the solution of the cost problem the control design software Matlab/Simulink is used. For this reason the state matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{F}$ were computed by the finite element package COSAR and transferred to Matlab/Simulink via a special data exchange interface. Based on these matrices Matlab/Simulink generates a linear, constant feedback gain matrix $\mathbf{L}$, which characterizes the actuator signal

$$\mathbf{u} = -\mathbf{L}\hat{\mathbf{z}}. \qquad (12)$$

The vector $\hat{\mathbf{z}}$ is the reconstruction of the state space vector based on the measured output.

Experimental Setup and Results

The setup for validation of the control approach described above is a clamped rectangular aluminium plate coupled with an acoustic cavity and with four piezoelectric patches attached to the plate surface. The proposed control concept was verified through Hardware-in-the-Loop experiments using a dSPACE control system. In the operation of the control system, a microphone was used to detect the sound pressure level in the cavity. A shaker was used to excite the plate with a harmonic force. The dSPACE system was set to sample the microphone signals at 10 kHz. The control board determined the necessary control outputs for the four actuators, based on the input data and implemented state space model.

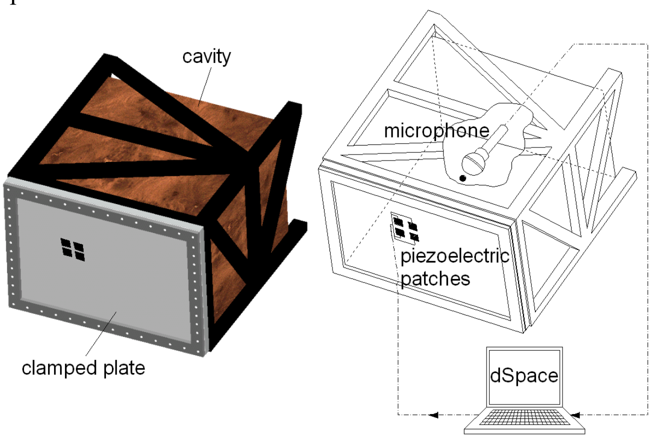

Figure 1: Experimental setup for measuring the uncontrolled and controlled sound level in the cavity of the smart acoustic box

During the tests, the microphone signals were monitored using a signal analyzer in dSPACE, which recorded the resulting pressure levels. The comparison of the experimental measured and simulated microphone response showed a good agreement.

References

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