Volumetric Reconstruction of Sound Intensity in Room Acoustics

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Introduction

A measurement-method for the volumetric reconstruction of the sound intensity is presented. The method offers the analysis of the anisotropic part of the room-impulse-response (RIR) throughout a large volume and avoids failures from local interferences of the sound-field with classical point-measurements. Field-holograms indicate the volumetric vector intensity as a function of frequency and time. Energy contributions to the RIR are easily tracked and the insight into the behavior of the sound-field is enhanced. Mid- and long-term applications lie in the field of room acoustical consultancy as well as physical and perceptual research.
The main issue of the presented method is the application of a spherical array that reconstructs the sound field inside a volume. The contribution at hand shows results from a fundamental research on volumetric reconstruction of sound intensity at our institutes, firstly, with a review of the fundamental algorithm and secondly, with a measurement example.

Fundamental Algorithm

In accordance with Nearfield Acoustical Holography [3], the sound field inside a volume is reconstructed from the information of the sound-field on its surface. The method is termed "volumetric reconstruction" [2]. For the application of a spherical array, which has the simplest closed surface in the three-dimensional space, an inner problem is formulated. Provided that the spherical volume V is source-free and the pressure information on a transparent and concentric surface S_0 ($r_{S_0} \leq r_V$) is known, the sound-pressure $p(r, \theta, \phi, \omega)$ in V can be fully reconstructed. The spherical coordinates are written as radius r, elevation θ and azimuth ϕ; ω denotes the angular frequency. To perform an extrapolation of field-quantities, the pressure-information on the surface S_0 is expanded into a series of Spherical Harmonics[1] (SH) $Y_n^m(\theta, \phi)$, which form a complete orthogonal set of solution of the wave-equation in θ and ϕ:

$$p(r_{S_0}, \theta, \phi, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} K_n^m(r_{S_0}, \omega) Y_n^m(\theta, \phi), \quad (1)$$

$n \in \mathbb{N}$ is the order and $m \in \mathbb{Z}$, $n \leq m \leq n$, is the mode of the corresponding SH. Using the orthogonality of the SH, the complex series coefficient K_n^m is calculated and multiplied with a quotient of spherical Besselfunctions $j_n(x)$ of first kind and order n to accomplish an extrapolation at different radii r within V:

$$K_n^m(r, \omega) = \frac{j_n(kr)}{j_n(kr_{S_0})} \int p(r_{S_0}, \theta, \phi, \omega) Y_n^m(\theta, \phi)^* \mathrm{d}\Omega, \quad (2)$$

in which $\mathbf{d}\Omega$ represents the solid angle, $(\bullet)^*$ denotes the complex conjugate, k is the wave number and c the speed of sound. Inserting the complex series coefficient K_n^m in (1), the sound pressure p at every point (r, θ, ϕ) in V can be calculated. For the spatial sampling on the surface S_0, we apply a Lebedev-quadrature, which is efficient with respect to the necessary number of recording positions at a given order n of the series expansion.
The computation of the vector intensity is based on a gradient method throughout the reconstructed pressure on a cubic grid. With Cartesian coordinates and unit vectors $\hat{e}$, the active and time averaged intensity is the result of:

$$\vec{I}(x, y, z, \omega) = \frac{1}{2} \Re[p^*(v_x \hat{e}_x + v_y \hat{e}_y + v_z \hat{e}_z)], \quad (3)$$

here $\Re(\bullet)$ is the real part. The contributions of the velocity vector are given from a finite difference approximation of Newtons second law. Considering the simplified case of two sound-pressure probes along the x-coordinate ($p(x_1, \omega)$ and $p(x_2, \omega)$), the sound-velocity is calculated with:

$$v_x(\omega) = \frac{1}{j\omega\rho_0 \Delta x} [p(x_1, \omega) - p(x_2, \omega)], \quad (4)$$

where $j = \sqrt{-1}$, ρ_0 is the medium density and Δx is the interspacing between the probes. By summing over all velocity vector contribution, the spatial direction of the local intensity flux is obtained. Applying consecutively (3) and executing this operation on the entire lattice of reconstructed pressure-scalars results in the volumetric vector intensity.

Intensity Measurement and Results

The numerical algorithm was tested and optimized in a simulation with a mirror image source model. Following this, a measurement with a scalable spherical two-microphone array was conducted in an anechoic room at the TU Delft. To arrange the simplified case of an RIR consisting of direct sound and one reflection, a single wall (made of medium density fiberboard, 2.4 x 2.88 m) was placed (see fig. 1) and a common mid-range speaker was turned towards it. The distance, the sound had to travel directly to the middle of the array was 4.5 m and via the reflection across the wall 13.2 m. The construction of the utilized spherical array is similar to the array presented in [1], except a second microphone rod for the benefit of

[1] Spherical Fourier Transform

measuring-time. The RIR on the surface of a sphere with radius $r = 0.5$ m was sampled with omnidirectional pressure microphones on a Lebedev-quadrature of order $n = 20$ with 590 recording positions. The configuration allows for a reconstruction of the sound-field up to 2.2 kHz. After recording the RIR on the surface the sphere, we

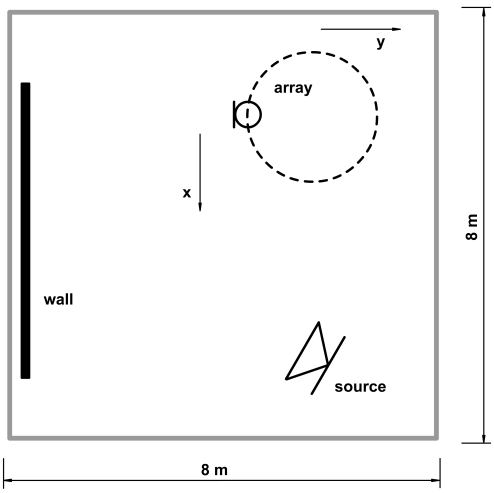

Figure 1: Sketch of measurement setup in an anechoic room, with one wooden wall inserted.

extracted direct sound and reflection, respectively, and transformed these in the frequency domain with a Fourier Transform. For discrete pressure-functions of frequency on the surface of the sphere, a Spherical Fourier Transformation and Bessel expansion (acc. to 2 and 1) were applied to reconstruct the sound-pressure on a cubic grid with a pressure-probe interspacing of 50 mm (Δx, Δy and Δz). This specific distance was chosen to yield accurate results up to 1.2 kHz without significant failures from the finite difference approximation in (4). Following (3), the gradient was applied to the field of scalars and multiplied with the locally averaged and complex conjugated pressure to arrive at the volumetric intensity. Figure 2 shows a hologram of the direct sound and fig. 3 of the reflection from the wall at an arbitrarily picked frequency of 550 Hz. The figures correctly indicate the direction of the intensity flux. Using the free-field relation, the approximated, mean intensity $I = p_{rms}^2/\rho_0 c$ was 65.3 dB (dB re 10^{-12}W/m^2) for the direct sound and 67.2 dB for the reflection at a frequency of 550 Hz. Also the reconstructed values showed the same order of magnitude, we encountered a considerable sensitivity towards sources of failures, e.g., measurement noise, mechanical imprecision and singularities in the radial solution of the wave equation, which altogether limited the correct reconstruction of the intensity flux for all possible frequencies.

Conclusions and Outlook

A method for the reconstruction of the volumetric intensity is presented. Field holograms qualitatively indicate the direction of incident sound intensity and overcome errors from interferences with single point-measurements. The method is sensitive to measurement-errors as well as

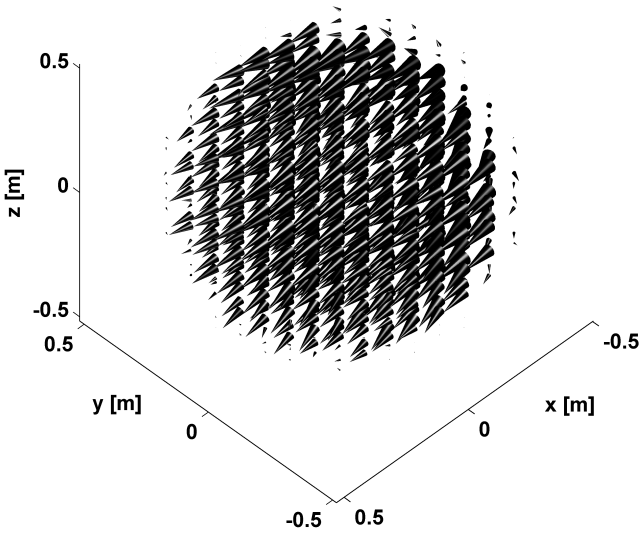

Figure 2: Volumetric intensity for the direct sound along the x-coordinate.

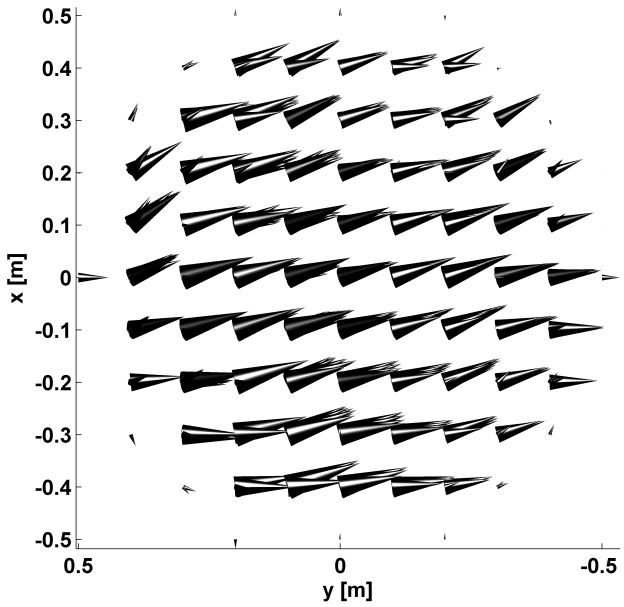

Figure 3: Top view on the volumetric intensity of the reflection at the wall, while the overall direction is correctly computed, local deviations indicate measurement and algorithmic errors.

singularities in the solution of the wave equation. Pursueing these problems is our future task.

References

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