Gittererzeugung für die FEM Analyse von akustischen Innenraummodellen
"Mesh generation for FEM calculations in Closed Acoustical Spaces"

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Introduction

An important and difficult step for simulating the sound inside acoustical spaces using FEM or BEM approaches is to find a proper discretization of a continuous domain. The process of dividing the domain into small and simpler elements like triangles and quadrilaterals in 2D and tetrahedrons, and hexahedrons in 3D is called mesh generation. In this paper we introduce a new scheme called "cutting plane algorithm" (based on cutting the polyhedron) in order to obtain an all hexahedral mesh. The cutting plane algorithm is inspired by the idea proposed by Chazelle [1]. It has been shown by Chazelle that the worst case time for obtaining $O(N^2)$ convex pieces from a polyhedron without holes is $O(nN^2(N+\log n))$, where n and N designate respectively the size of the input and the number of reflex angles or notches into the polyhedron. Reflex edges/Notches are the edges whose adjacent faces make an interior dihedral angle greater than 180 degree with each other. Furthermore, it is also mentioned by Chazelle that repeating the cutting process on each remaining non-convex parts will eventually produce a convex decomposition in a finite number of steps.

In this paper, apart from cutting notches, we are removing multivalent vertices (valence greater than three) as well. The *valence* of a vertex is defined as the number of edges connected to it. This would further increase the worst case time calculations. We do not perform its worst case calculations but as a first step, our main emphasis is to investigate its practical applicability in room acoustics. Special cutting schemes are suggested for architectural designs where balconies, domes etc. are very common.

Cutting Plane Algorithm

Any architectural closed space can be treated as a polyhedron of arbitrary shape. As it is very difficult to obtain an automated mesh for complicated shapes, the cutting plane algorithm aims at reducing the complexity of the original polyhedron by applying a series of cuts unless one obtains only simple shaped polyhedrons. Two of such complexities which the cutting plane algorithm tries to remove are:

1. Notches / Reflex Edges : Edges whose adjacent faces make an interior dihedral angle greater than 180 degree with each other are called notches or reflex edges. In the illustration (figure 1a), one can see there is one such notch (marked with thick/bold line). A **cut** is defined to be the procedure of cutting the polyhedron along such notches so as to remove the concavity in the geometry. For example, figure 1b shows two polyhedrons (**P1** and **P2**) obtained after cutting along the notch.

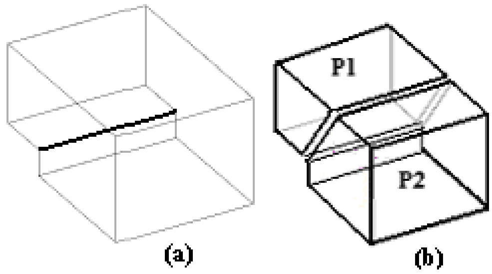

Figure 1: L-Shaped room with one Notch

2. Multivalent Vertices: Next, one should observe that even after cutting notches, we might not be able to obtain all trivalent polyhedrons. For example, see figure 2a. Here the topmost vertex is four valent. Hence we need to find some routine to decrease the valence of such vertices. For such multivalent vertices, valence can be decreased by cutting along two adjacent edges such that at least two other adjacent edges are on the opposite side of the cutting plane. See figure 2b. Now the topmost vertex is 3-valent in both polyhedrons.

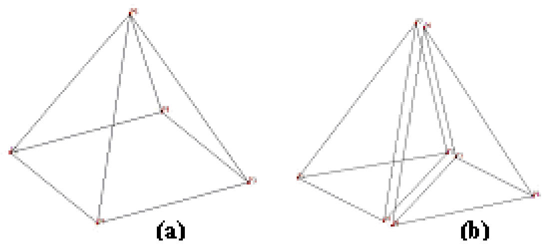

Figure 2: Multivalent Vertex cut

Next, after obtaining convex and trivalent polyhedrons, one can apply mid-point subdivision scheme [2] on each polyhedron to obtain a hexahedral mesh.

The chart (figure 3) further explains the basic methodology adopted for mesh generation using the cutting plane algorithm. The chart explains that the cutting plane algorithm starts with examining of the geometry type of the given room. Firstly, on the basis of geometry, it suggests some initial cutting directions to minimize the total number of cuts. For instance, removing consecutive or continuously attached notches all at once or dealing with some specific room structures like balconies, domes, stairs, holes etc. If initially, one does not treat them in a proper manner, a random cutting scheme could destroy their identity and create bad shaped polyhedrons. So first we cut the polyhedron along those predetermined specific cuts. Secondly, we check the single notches one by one and remove them. Then in the third step, we remove multivalent vertices one by one. Fourth step is mainly to check and cut to improve aspect ratio of elements and then finally in the fifth step, the simple shaped polyhedrons are then meshed using midpoint division scheme to obtain hexahedral mesh.

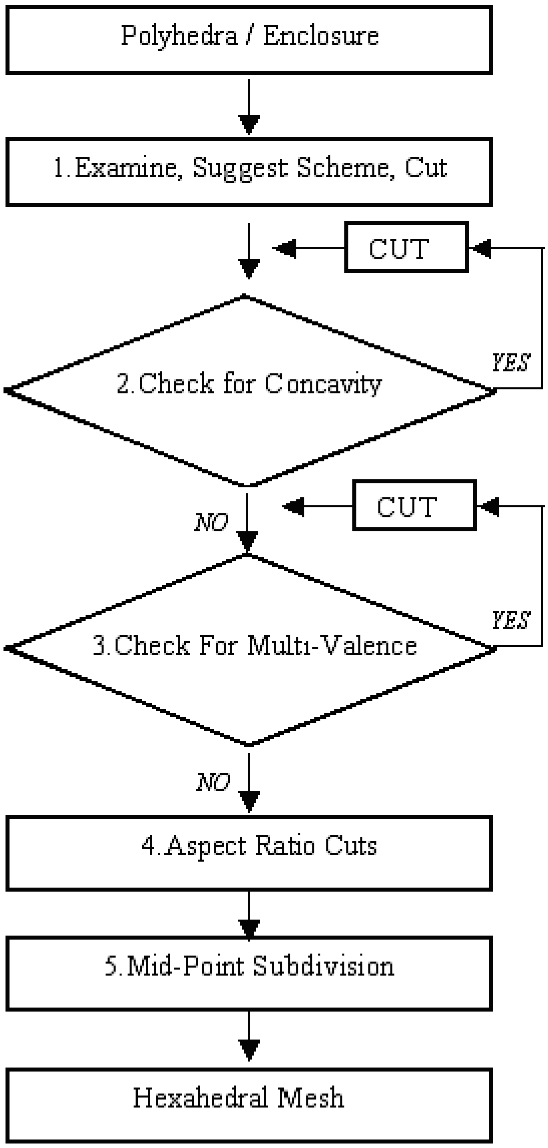

Figure 3: Methodology

Here, one can observe that the main algorithm is the cut procedure. No matter whether it is cutting specific notches, normal notches or multivalent vertices, the basic cut procedure is same for all of them only the definition of cutting plane changes.

Real World Examples

In order to demonstrate the applicability and advantages of our algorithm, we show how one can apply it to more complicated and real-life typical rooms where much acoustical considerations are needed. Consider, the Church example (fig. 4a). This room is a combination of consecutive notches and multivalent vertices. Figure 4b shows the mesh after applying the proper sequence of cutting planes.

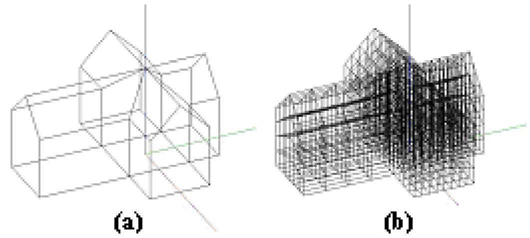

Figure 4: Church

Next, consider the Gothic Dome example (figure 5a). It is again a combination of continuous notches in addition to a half dome like structure. First the dome was removed and then the consecutive notches were cut sequentially in order to obtain the mesh shown in figure 5b.

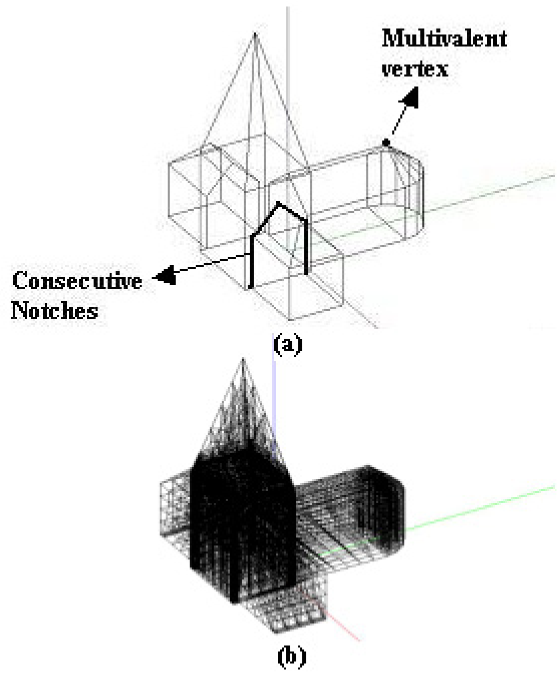

Figure 5: Gothic Dome

From the above illustrated examples, it is quite clear that using our approach many such complicated real-life geometry examples can be meshed. Afterwards, these meshes can be used to simulate the sound field in such spaces using finite element method [3].

References

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