Notes on Rendering Focused Directional Virtual Sound Sources in Wave Field Synthesis

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Introduction

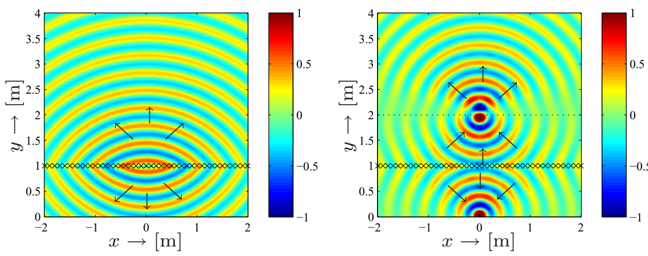
Wave field synthesis (WFS) is a technique aiming at the reproduction of a desired sound field. It is based on the Kirchhoff-Helmholtz integral formulated for interior problems [1]. This implies that WFS is per se only capable of recreating the wave field generated by events outside the listening area. In particular, the traditional WFS literature (e.g. [2]) focuses on the reproduction of the sound field of virtual point sources which are positioned outside the listening area ('behind' the loudspeakers). For both restrictions of the traditional WFS literature - virtual *point* sources *outside* the listening area - extensions have been proposed. In this paper, we briefly review the strategies proposed and comment on the particularities arising in the combination of the extensions, i.e. the reproduction of complex (directional/spatially extended) virtual sources positioned inside the listening area ('in front of' the loudspeakers). For simplicity, we will restrict the treatment to two-dimensional reproduction.

Wave field synthesis

Although WFS is generally not restricted to a particular loudspeaker setup, we assume a continuous linear distribution of secondary line sources for convenience. The theoretical basis of WFS employing this setup is given by the two-dimensional Rayleigh I integral [2]. It states that a linear distribution of monopole line sources is capable of reproducing a desired wave field (a virtual source) in one of the half planes defined by the secondary source distribution. This half-space has to be free of sources. The wave field in the other half (where the virtual source is situated) is a mirrored copy of the desired wave field.
For convenience the secondary source array is assumed to be parallel to the x-axis at $y = y_0$ and to be of infinite length. The virtual source is situated in the origin of the coordinate system. Any other situation may be treated by an appropriate rotation and/or translation of the coordinate system. The listening area is chosen to be at $y > y_0$.
The two-dimensional Rayleigh I integral determines the sound pressure $P(\mathbf{x}, \omega)$ created by such a setup reading

$$P(\mathbf{x}, \omega) = -\int_{-\infty}^{\infty} \underbrace{2 \frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}, \omega)\big|_{\mathbf{x}=\mathbf{x}_0}}_{D(\mathbf{x}_0, \omega)} \underbrace{\frac{j}{4} H_0^{(2)} \left(\frac{\omega}{c} |\mathbf{x} - \mathbf{x}_0| \right)}_{G_{2D}(\mathbf{x}-\mathbf{x}_0, \omega)} dx_0 \,, \tag{1}$$

where $S(\mathbf{x}, \omega)$ denotes the sound field of the virtual sound source, $G_{2D}(\mathbf{x} - \mathbf{x}_0, \omega)$ the 2D free-field Green's function representing the secondary sources, $H_0^{(2)}(\cdot)$ the zeroth order Hankel function of second kind, and $\frac{\partial}{\partial \mathbf{n}}$ the gradient in the direction normal to the secondary source distribution. $\mathbf{x}$ is a point inside the half plane where the sound field is recreated. $D(\mathbf{x}_0, \omega)$ is the driving signal for a secondary source situated at $\mathbf{x}_0 = [x_0 \; y_0]^T$. The Green's function $G_{2D}(\mathbf{x} - \mathbf{x}_0, \omega)$ may be interpreted as the sound field of a line source perpendicular to the x-y-plane intersecting it at $\mathbf{x}_0$.
Typical WFS systems employ loudspeakers with closed cabinets as secondary sources. These approximately exhibit the characteristics of acoustic point sources. This mismatch in source types produces various artifacts which can be compensated for to a limited extend [2]. We will restrict our descriptions to the artifact free case of secondary line sources for convenience. However, our results also hold for secondary point sources within the limitations of the compensation of the mismatch artifacts.



(a) Non-focused monopole source. (b) Focused monopole source.

Figure 1: WFS reproducing a virtual monopole source. The marks indicate the positions of the secondary sources. The arrows indicate the local propagation direction of the reproduced wave field.

Focusing of virtual sound sources

Due to causality restrictions it is not possible to perfectly reproduce the wave field of a sound source situated in the listening area. A sound source is an entity which radiates energy. However, by means of the employment of secondary sources (i.e. loudspeakers), it is only possible to radiate energy (sound waves) into the listening area (away from the secondary sources).
In order to evoke the perception of a virtual source inside the listening area, a wave field converging towards a focus point can be reproduced. As a consequence of causality, the wave field diverges after having passed the focus point (see also [3]). A listener positioned in the diverging part of the wave field perceives a virtual source

at the position of the focus point. A listener positioned in the converging part of the wave field will experience of confusing perception due to contradictory localization cues. Note that it is not possible to focus a wave field such that it exclusively diverges in a volume greater than one half-space. The boundary of this half-space includes the position of the focus point.

The reproduction of such a wave field converging in a given volume and diverging in another volume is referred to as reproduction of a *focused virtual source*.

In order to reproduce a converging wave field, the time-reversal principle is applied in WFS [2]. Note that a converging wave field is essentially a time-reversed diverging one. One proceeds as follows: First, a virtual source is created at the position of the intended focused source but its radiated wave field is recreated behind the loudspeaker array (i.e. at $y < y_0$ in figure 1(a)). Then, the loudspeaker driving functions are time reversed. This results in a wave field that converges towards the location of the virtual source and then diverges and makes up the desired wave field. Note that the implementation of focused virtual sources requires to anticipate the source input signal (i.e. to apply a negative delay). In order to fulfill causality, the source input signal has to be pre-delayed. The anticipation is then performed virtually within this pre-delay.

From the symmetry of the temporal Fourier transformation of purely real time-domain signals we know that

$$f(-t) \quad \circ\!\!-\!\!\!-\!\!\bullet \quad F(-j\omega) \;=\; F^*(j\omega) \tag{2}$$

holds [4]. Thus, the temporal spectrum of the driving function for a focused virtual source can be yielded by a complex conjugation of the temporal spectrum of the driving function of the virtual source.

The spatio-temporal transfer function of a two-dimensional point source situated in the origin of the coordinate system is given by

$$S_{\text{point}}(\mathbf{x},\omega) = H_0^{(2)}\left(\frac{\omega}{c}|\mathbf{x}|\right) \; . \tag{3}$$

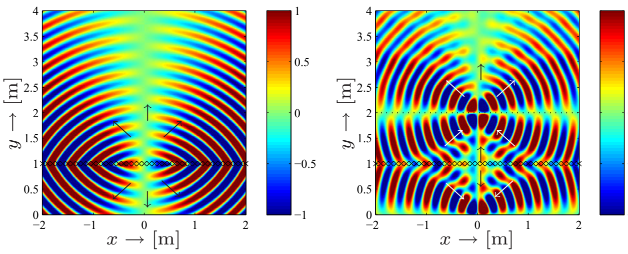
The according WFS driving function can be found in [1]. Figure 1 depicts a simulation of the wave field of a WFS system reproducing a non-focused as well as a focused virtual monopole source. In figure 1(b), the dashed line indicates the boundary of the half-space where the reproduced wave field diverges.

The spatio-temporal transfer function of a two-dimensional complex source, i.e. a directional or spatially extended source, can be conveniently described by a circular harmonics expansion reading

$$S_{\text{compl}}(\mathbf{x},\omega) = \sum_{\nu=-\infty}^{\infty} \breve{S}^{(2)}(\nu,\omega) H_\nu^{(2)}\left(\frac{\omega}{c}|\mathbf{x}|\right) e^{j\nu\alpha} \; , \tag{4}$$

when the source is positioned in the origin of the coordinate system. $\breve{S}^{(2)}(\nu,\omega)$ denote the circular harmonics expansion coefficients. The according WFS driving function can be found in [5]. Figure 2 depicts a simulation of the wave field of a WFS system reproducing a non-focused as well as a focused virtual dipole. A dipole was chosen as sample complex source since it allows to easily identify the desired components of the reproduced wave field as well as artifacts introduced by the reproduction system.

However, the following particularity arises in the reproduction of focused complex sources: As described above, the first step towards the focusing of a virtual source is to model a virtual source at the position of the intended focused source but radiating into the exterior of the listening area. The virtual source has a given orientation, say towards the loudspeakers. When the time is reversed in the driving functions, the wave field virtually radiated by the source towards the loudspeakers converges towards the location and diverges from thereon. As a consequence, the focused virtual source will appear to the listener positioned in the diverging part of the reproduced wave field as being oriented away from the secondary sources although it was originally positioned facing the secondary sources. Therefore, a virtual complex source has to be rotated by 180° when it is focused in order to appear to the listener with the correct orientation.



(a) Non-focused dipole. (b) Focused dipole.

Figure 2: WFS reproducing a virtual dipole. The marks indicate the positions of the secondary sources. The arrows indicate the local propagation direction of the reproduced wave field.

Conclusions

When the secondary source driving functions for WFS reproduction of focused virtual complex sources are derived by employing the time-reversal principle, care has to be taken that the orientation of the reproduced virtual source appears as intended. The listening area is limited to one half-space whose boundary includes the position of the focused source.

References

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