Investigation of Spatial Aliasing Artifacts of Wave Field Synthesis in the Temporal Domain

Sascha Spors
Deutsche Telekom Laboratories, Technische Universität Berlin, Ernst-Reuter-Platz 7, 10587 Berlin, Germany
Email: Sascha.Spors@telekom.de

Introduction

Wave field synthesis (WFS) is a spatial sound reproduction technique that uses a high number of densely placed loudspeakers (secondary sources). This implies a spatial sampling process that may lead to aliasing artifacts. In the past, the spatial aliasing artifacts of WFS have mainly been investigated for the reproduction of monochromatic signals, and hence in the frequency domain. This contribution analyzes the aliasing artifacts for broadband signals. The derived results give insights into the spatio-temporal structure of spatial aliasing for e. g. transient signals.

Wave Field Synthesis

WFS aims to physically recreate the wave field of a desired virtual source $S(\mathbf{x}, \omega)$. Typical implementations are restricted to the reproduction in a plane only using (piecewise) linear loudspeaker arrays. The theoretical basis for linear secondary source contours is given by the Rayleigh I integral [1, 2]. Without loss of generality the geometry depicted in Fig. 1 is assumed: A linear secondary source distribution which is located on the x-axis ($y = 0$) of a Cartesian coordinate system. The Rayleigh I integral reads

$$P(\mathbf{x}, \omega) = -\int_{-\infty}^{\infty} \frac{j}{4} H_0^{(2)}(\frac{\omega}{c}\,|\mathbf{x} - \mathbf{x}_0|)\; \underbrace{2\frac{\partial}{\partial \mathbf{n}} S(\mathbf{x}_0, \omega)}_{D(\mathbf{x}_0,\omega)}\; dx_0 \;, \tag{1}$$

where $\mathbf{x}_0 = [\; x_0 \quad 0 \;]^T$, $\frac{\partial}{\partial \mathbf{n}}$ denotes the directional gradient in direction of the normal vector $\mathbf{n} = [\; 0 \quad 1 \;]^T$ and $H_0^{(2)}(\cdot)$ the zeroth-order Hankel function of second kind. The Hankel function in (1) can be interpreted as the field of a line source intersecting the listening area at the position $\mathbf{x}_0$ [2]. Two-dimensional WFS systems approximate the required secondary line sources by point sources. However, the resulting artifacts (e. g. amplitude errors) will have no influence on the analysis of spatial aliasing.
The Rayleigh I integral states that a linear distribution of monopole line sources is capable of exactly reproducing the wave field of the virtual source in one of the half planes defined by the linear secondary source distribution (e. g. $y > 0$). The wave field in the other half plane is a mirrored version of the desired one. The terms involving the strength of the secondary sources (driving signal) are abbreviated by $D(\mathbf{x}_0, \omega)$.
Eq. (1) can be interpreted as a spatial convolution along the x-axis [3]. This interpretation is the basis for deriving the spatial aliasing artifacts in the following section.

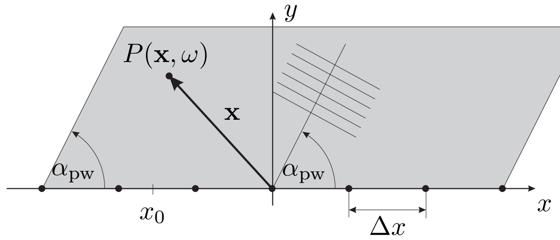


Figure 1: Geometry used to derive the sampling artifacts for linear loudspeaker arrays.

Monochromatic Plane Waves

A detailed analysis of spatial sampling for linear WFS systems was already presented in [3]. However, this analysis was confined to monochromatic plane waves. This section briefly summarizes the results derived in [3]. The driving signal $D(x, \omega)$ is sampled at equidistant positions, in order to model the effect of a spatially discrete secondary source distribution. The process of sampling can be described mathematically by multiplying the continuous driving signal with a series of Dirac functions

$$D_S(x, \omega) = D(x, \omega) \cdot \frac{1}{\Delta x} \sum_{\mu=-\infty}^{\infty} \delta(x - \Delta x \mu) \;, \tag{2}$$

where $D_S(x, \omega)$ denotes the sampled driving signal and Δx the distance between the sampling positions. These positions are indicated in Fig. 1 by the dots •. Introducing (2) into (1) allows to derive the wave field reproduced by a discrete secondary source distribution. Such a formulation is useful for numerical simulation, however, no insights into the spatial structure of spatial aliasing artifacts can be gained.
An analysis in the spatial frequency domain by way of a spatial Fourier transform has proven to gain valuable insight into the spatial structure of spatial aliasing artifacts. Spatial sampling of the driving signal, as given by (2), results in repetitions of its spatial spectrum. The spatial convolution in (1) is resolved into a multiplication of the spatial spectrums of the driving signal and the secondary sources. Hence, spatial sampling of the secondary source distribution can be understood as a sampling and interpolation process. The interpolator is given by the characteristics of the secondary sources. These characteristics are, in general, given by the physical realization of the secondary sources and can be only be influenced to some degree. An anti-aliasing condition has been derived for monochromatic plane waves [3]. If the anti-aliasing condition is not fulfilled, aliasing artifacts will be present in the reproduced wave field. These artifacts are a superposition of plane waves with different incidence angles than the desired plane wave.

Extension to Broadband Signals

The spatial sampling of the driving signal results in an infinite number of equidistant repetitions in the spatial frequency domain. However, not all of these spectral repetitions will be reproduced due to the properties of the secondary sources. A formulation of the reproduced wave field in the spatial frequency domain, as given in [3], allows to identify the actually reproduced plane wave contributions and their characteristics. This is the basis for extending the theory in [3] to broadband signals.
Only a subset η of all possible spectral repetitions will be present in the reproduced wave field. This subset includes all $\eta \in \mathbb{Z}$ for which the following condition holds

$$\left| \frac{1}{\Delta x}\eta + \frac{f_{\mathrm{pw}}}{c} \cos \alpha_{\mathrm{pw}} \right| \leq \frac{f_{\mathrm{pw}}}{c} \,, \tag{3}$$

where f_{pw} denotes the frequency of the desired plane wave. Using the subset η defined by Eq. (3), the incidence angles $\alpha_{\mathrm{pw},\eta}$ of the plane wave contributions of the reproduced wave field can be derived as

$$\cos \alpha_{\mathrm{pw},\eta} = \frac{c}{\Delta x \; f_{\mathrm{pw}}}\eta + \cos \alpha_{\mathrm{pw}} \,, \tag{4}$$

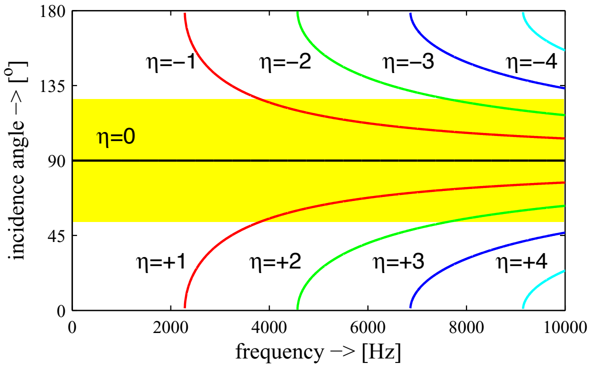
and their amplitudes $a_{\mathrm{pw},\eta}$ as

$$a_{\mathrm{pw},\eta} = \frac{\sin \alpha_{\mathrm{pw}}}{\sqrt{1 - (\frac{c}{\Delta x \; f_{\mathrm{pw}}}\eta + \cos \alpha_{\mathrm{pw}})^2}} \,. \tag{5}$$

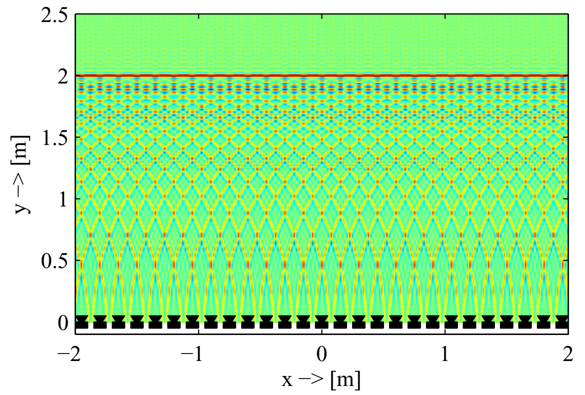
Two conclusions can be drawn from Eq. (4) and (5): first the incidence angle and amplitude of the desired plane wave ($\eta = 0$) stays constant with frequency, and second the incidence angles and amplitudes of the plane waves constituting aliasing ($\eta \neq 0$) vary with frequency.
Eq. (4) and (5) can be used to reconstruct the wave field reproduced by a linear WFS systems as superposition of plane waves with frequency dependent incidence angles and amplitudes.
For the reproduction of plane waves and a reasonable aperture, the effect of truncation can be approximated quite well by simple geometric means. This approximation states that a plane wave will be reproduced only in a tilted rectangular area in front of the array, whose width is equivalent to the aperture of the array and length is infinite (see gray area in Fig. 1). As a consequence to this limited reproduction area, the aliasing artifacts depend on the listener position for finite length WFS systems.

Results

The wave field generated by a linear WFS system with a secondary source distance of $\Delta x = 10$ cm reproducing a band-limited plane wave with incidence angle $\alpha_{\mathrm{pw}} = 90^{\mathrm{o}}$ and temporal bandwidth $B = 10$ kHz is investigated to illustrate the theory presented.
Figure 2(a) shows the incidence angles of the plane wave contributions present in the reproduced wave field. The effect of truncation is illustrated by the (yellow) shaded area for a total length $L = 3$ m of the loudspeaker array and the listener position $\mathbf{x} = [\; 0 \quad 2 \;]^T$ m. Only those plane wave contributions that lie within the shaded area



(a) incidence angle of plane wave contributions



(b) superposition of plane wave contributions

Figure 2: Reproduction of a bandlimited plane wave ($B =$ 10 kHz, $\alpha_{\mathrm{pw}} = 90^{\mathrm{o}}$) on a linear WFS system ($\Delta x = 10$ cm).

will be present at that particular listener position. Note the increased aliasing frequency in this case. Figure 2(b) illustrates the reproduced wave field when superimposing the desired and all aliased plane wave contributions. The results match closely with a numerical simulation of the reproduced wave field. However, such a simulation would not allow the interpretation of aliasing as superposition of plane waves.

Conclusion

This paper analyzes the spatio-temporal structure of spatial aliasing artifacts for the reproduction of broadband plane waves using linear WFS systems. It was shown that spatial aliasing constitutes a superposition of plane waves with frequency dependent incidence angles and amplitudes. The presented theory allows a strict distinction between spatial aliasing and truncation artifacts.

References

[1] A.J. Berkhout, D. de Vries, and P. Vogel. Acoustic control by wave field synthesis. *Journal of the Acoustic Society of America*, 93(5):2764–2778, May 1993.

[2] E.G. Williams. *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography*. Academic Press, 1999.

[3] S. Spors. Spatial aliasing artifacts produced by linear loudspeaker arrays used for wave field synthesis. In *Int. Symp. on Control, Communications, and Signal Processing*, Marrakech, Morocco, March 2006.