Evaluation of Active Noise Control Strategies

Thomas Kletschkowski[1], Delf Sachau[2]

[1] Helmut-Schmidt-University/UnBwH, Mechatronics, 22039 Hamburg, E-Mail: Kletsch@hsu-hh.de

[2] Helmut-Schmidt-University/UnBwH, Mechatronics, 22039 Hamburg, E-Mail: Sachau@hsu-hh.de

Introduction

Different approaches can be applied for active control of interior noise. If the anti-noise is generated by loudspeakers, the approach is known as active noise control (ANC). If an active control system consists of structural actuators and acoustic sensors, active structural acoustic control (ASAC) is applied. If structural actuators as well as structural sensor are used, active vibration control (AVC) is applied. However, if it is impossible to block a specific transmission path by AVC or to reduce noise radiation into the enclosure by ASAC, ANC is in many situations a valid approach that can directly be applied to the acoustic field by causing only minor retroaction on the surrounding structure. This statement will be motivated by a simplified analysis of an air filled duct that is coupled to a vibrating structure. To evaluate different ANC strategies, a close form solution will be derived in frequency domain using a travelling wave model. The advantages and disadvantages of different control strategies such as minimization of pressure and pressure gradient will be discussed with respect to the reduction of the potential energy density of the benchmark system.

Why to use ANC?

The application of ANC can yield to a reduced control effort and causes therefore only minor retroaction on the surrounding structure. This statement can be motivated by a simplified analysis of an air filled duct (Fig. 1 (top)) with length L and cross section S, fluid density ρ, and speed of sound c. This duct is coupled to a vibrating structure with mass m_s and stiffness c_s that may at $x = 0$ be excited by a harmonic excitation force $F = \hat{F}\cos(\Omega t)$.

For frequencies up to the first acoustic mode the air filled volume may be represented by two discrete springs each with stiffness $c_l/2 = \rho c^2 S/(2L)$ and one discrete mass with weight $m_l = \rho SL$ as shown in Fig 1 (middle). Damping effects may be excluded.

Structural actuation at the vibrating structure itself is modelled by the control force F_s that can be generated by an AVC or by an ASAC approach. Acoustic actuation inside the cavity is represented by F_{ANC}. The equation of motion for the controlled system reads

$$\begin{pmatrix} m_s & 0 \\ 0 & m_l \end{pmatrix}\begin{pmatrix} \ddot{u}_s \\ \ddot{u}_l \end{pmatrix} + \begin{pmatrix} c_s + c_l/2 & -c_l/2 \\ -c_l/2 & c_l \end{pmatrix}\begin{pmatrix} u_s \\ u_l \end{pmatrix} = \begin{pmatrix} F + F_s \\ F_{ANC} \end{pmatrix}. \quad (1)$$

It follows that a structural control force $F_s = -F$ is required to cancel the external load F with AVC or ASAC. Solving (1) for harmonic excitation leads to an optimal acoustic actuation of

$F_{ANC} = -\left[2c\left(1-\Omega^2/\omega_s^2\right)+1\right]^{-1} F$, where $\omega_s^2 = c_s/m_s$ and $c = c_s/c_l$ that is required to cancel the force in the right spring (e. g. the sound pressure in the right part of the duct). The introduction of a logarithmic measure such as $A_f = -20\log_{10}\left(\left|2c\left(1-\Omega^2/\omega_s^2\right)+1\right|\right)$ allows for a graphical representation (Fig 1 (bottom)) of the relation between magnitudes of F_{ANC} and F.

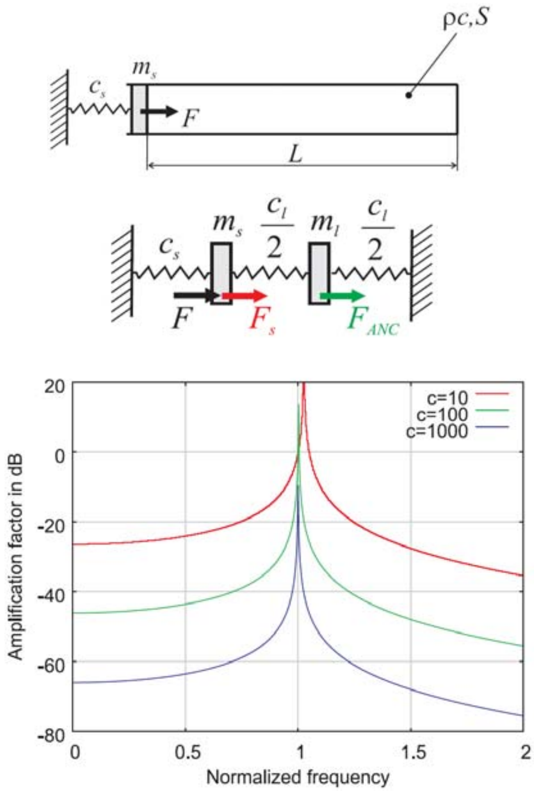

Figure 1: Air filled duct coupled to a vibrating structure with excitation force F (top), representation of air filled duct by two discrete springs, one discrete mass and control forces actuation is represented by F_s and F_{ANC} (middle), and relation between acoustic actuation and external load (bottom). Curves peak in resonances $\Omega^2/\omega_s^2 = 1 + \frac{1}{2c}$, because damping is not taken into account.

Fig. 1 (bottom) in which A_f is plotted against the normalized frequency $\tilde{\omega} = \Omega/\omega_s$ clarifies that the control effort can significantly reduced1, if ANC is applied to interior noise problems. This statement holds, because the surrounding structure is (in general) stiff compared to the acoustic field that is enclosed in the cavity.

Evaluation of ANC Strategies

Description and Solution of Benchmark Problem

The benchmark problem may be given by an air filled finite acoustic duct with length L and cross section S that is at both ends terminated by loudspeakers. These loudspeakers are seen as ideal volume velocity sources with source strengths q_p at $x = 0$ (position of primary source), and q_s at $x=L$ (position of cancelling source). Assuming harmonic fluctuation of the source strengths, the problem is described by the Helmholtz equation $\Delta p(x)+(\omega/c)\,p(x)=0$ and the Neumann boundary conditions at $x=0$ and $x = L$. Applying a travelling wave model, the acoustic pressure is given by

$$p(x)=\frac{\rho c}{S}\frac{\left(e^{jk^*L}q_p+q_s\right)e^{-jk^*x}+\left(e^{-jk^*L}q_p+q_s\right)e^{jk^*x}}{e^{jk^*L}-e^{jk^*L}}, \qquad (3)$$

where $k^*=(\omega/c)(1-j\xi)$ is the complex wave number. Assuming small values of the damping ratio ξ, the damping terms are approximated by a fist order series expansion of the exponential functions such as $e^x\approx 1+x$.

Comparison of ANC Strategies

The amount of active noise reduction (NR) as well as the control effort of the following control strategies was analysed using analytical solutions derived from the benchmark model by evaluation of cost functions such as:

- **S1** - Global control of acoustic potential energy (Epot_global) $J_{S1}:=\rho^{-1}c^{-2}\int_{x=0}^{x=L}\left|p(x)\right|^2\rightarrow Min,$
- **S2**: Local control of potential acoustical energy Density (Epot_local) $J_{S2}:=\left|p(x=L)\right|^2\rightarrow Min,$
- **S3**: Minimization of total power input (TPI) $J_{S3}:=\mathrm{Re}\left\{p^*(x=0)q_p+p^*(x=L)q_s\right\}\rightarrow Min,$
- **S4**: Local control of sound pressure and particle velocity (E_local) $J_{S4}:=\tfrac{1}{2}\rho^{-1}c^{-2}\left|p(x=L)\right|^2+\tfrac{1}{4}\rho\left|u(x=L)\right|^2\rightarrow Min,$
- **S5**: Maximization of power absorption (PowerAbs) $J_{S5}:=\mathrm{Re}\left\{p^*(x=L)q_s\right\}\rightarrow Max,$
- **S6**: Maximum acoustic absorption (AcousticAbs) $$J_{S6}:=\frac{\rho c}{S}\left|\frac{\left(e^{-jk^*L}q_p+q_s\right)e^{jk^*x}}{e^{jk^*L}-e^{jk^*L}}\right|^2\rightarrow Min.$$

Quadratic optimization of cost functions yields to optimal source strengths $q_{J_{Si}}^{opt}(j\Omega)=V_{J_{Si}}(j\Omega)q_p(j\Omega)$ and allows for calculating f, the ratio between the global acoustic potential energy of the optimal controlled and the global acoustic potential energy of the uncontrolled system. The low frequency noise reduction for each ANC strategy was calculated by integration of f over the investigated frequency range such as

$$NR=10\log_{10}\left(\int_{\zeta=0}^{\zeta=2.75}f\,d\zeta\right)\quad\text{with}\quad\zeta=\frac{2\pi c}{\Omega l}. \qquad (2)$$

Furthermore, the low frequency control effort was calculated such as

$$L_{q_s}=10\log_{10}\left(\int_{\zeta=0}^{\zeta=2.75}\frac{\left|q_{J_{Si}}^{opt}\right|^2}{q_0^2}d\zeta\right)\quad\text{with}\quad q_0^2=5\cdot 10^{-8}\,\frac{\mathrm{m}^3}{\mathrm{s}}. \qquad (3)$$

The frequency range given in Eqn. (2) and (3) includes the first six acoustic modes.

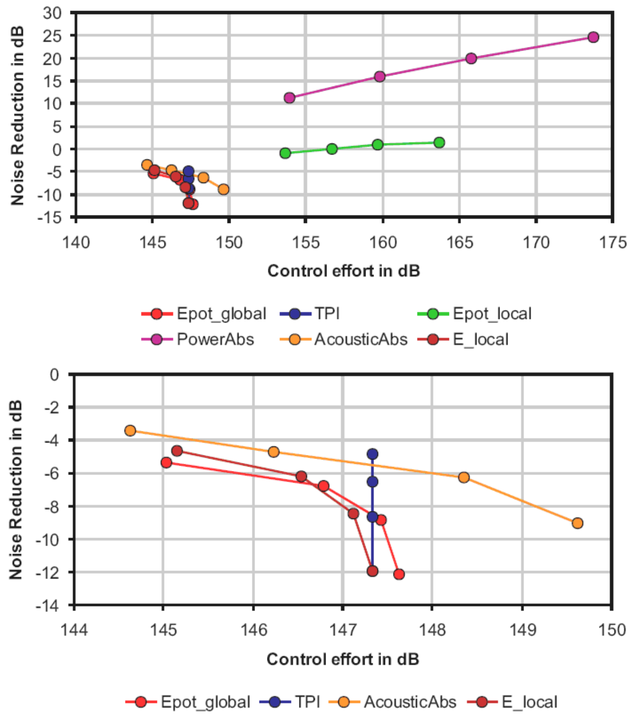

Figure 2: Comparison of integral noise reduction and integral control effort realized by different control strategies (red: global minimization of acoustic potential energy, green: local minimization of potential energy, blue: minimization of total power input, purple: maximization of power absorption, yellow ochre: maximization of acoustic absorption, brown: local minimization of energy density) for frequencies below the seventh acoustic mode for different values of viscous damping that has been increased from ξ = 0.01 (blue curve: bottom, all other curves right) over ξ = 0.025 and ξ = 0.05 to ξ = 0.1 (blue curve: top, all other curves left). Primary source strength is 150.4 dB.

The results are shown in Fig. 2. It turned out that all strategies can be applied to suppress acoustic resonances, but the control strategies S2 and S5 must not be applied outside resonances. The control profit is maximized by S1, and except for ξ = 0.01 this strategy also yields to the smallest control effort. However, it should be notices that nearly the same results can be obtained, if S4 – a local concept – is applied. A more details will be published in [1].

References

[1] Kletschkowski, T.: Adaptive Control of Low Frequency Interior Noise. Habilitation, submitted to the Helmut-Schmidt-University/ University of the Federal Armed Forces in December 2009.