Numerical investigation of transmission, reflection and scattering of sound waves through a blade row

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Introduction

The authors present a method to simulate the sound transmission through a 2D blade row by using the DLR in-house CAA-tool PIANO (Ref. [1]). The work of Kaji and Okazaki [2, 3] is used as a theoretical background. The numerical results show a good agreement with the theory and lead to further insight into the problematics of sound transmission and scattering by a blade row.

Method

The simulations are performed in two dimensional space with infinitely thin blades and no flow (see Fig. 1). The geometrical variables s, c and β represent the blade spacing, the chord length and the stagger angle, respectively. The angle of the incident plane wave is denoted α_0 while $\alpha^{\pm}$ describes the transmission and reflection. The domain is periodic in the cascade direction y and contains a suitable number of blades B. Slip-wall boundary conditions (BCs) are applied on the blade surfaces.

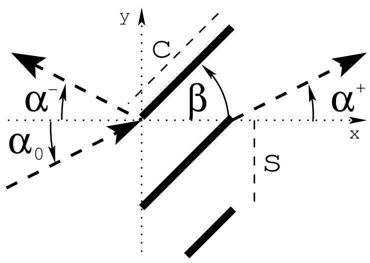


Figure 1: Schematic description of the cascade.

Due to the periodic BCs, a spectrum of discrete propagation angles $\hat{\alpha}_m$ is obtained. These angles are linked to the vertical extent of the computational domain Bs by

$$\hat{\alpha}_m = \arcsin\left(\frac{m\lambda}{Bs}\right), \tag{1}$$

with the wavelength λ and $m = 0, \pm1, \pm2, \ldots, \pm N_m$, where $N_m = \frac{Bs}{\lambda}$ represents the maximum number of angles in the spectrum. These angles are called geometrical modes $\hat{\alpha}_m$ of the domain.

The sound field is decomposed upstream and downstream of the blade row into the complex amplitudes A_m of each mode m:

$$p(x, y) = \sum_m A_m e^{-i(k_{x,m}x + k_{y,m}y)}, \tag{2}$$

where the vertical wavenumber k_y is fixed by the periodic BCs and the axial wavenumber k_x is deduced from the dispersion relation:

$$k_{y,m} = \frac{2\pi m}{Bs}, \qquad k_{x,m} = \sqrt{k^2 - k_{y,m}^2}, \tag{3}$$

with k the free-field wavenumber.

It can be observed in Fig. 2 that running simulations on multiple segments for a wave that is periodic on a single segment gives the exact same results. This indicates that the scattered modes α_ν fulfill the same periodic BCs as the incidence mode α_0. Thus a coupling of the the geometric modes $\hat{\alpha}_m$ and the scattered modes α_ν can be found as follows.

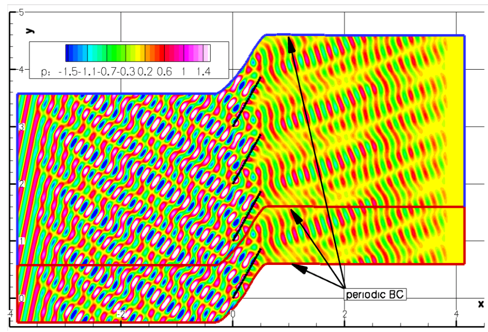


Figure 2: Comparison between the solutions on one segment (red) and three segments (blue) at same periodic conditions ($\beta = 60°$, $s = 1c$, $\lambda = 0.21c$, $\alpha_0 = -12.1°$).

Sound propagation through a blade row is divided into sub- and super-resonance [3]. The latter case describes the state where scattering occurs. Kaji formulates the condition for existence of a scattered mode α_ν at super-resonance without flow as:

$$\frac{s}{\lambda} \geq \nu \frac{\sin(\alpha_0) + \mathrm{sgn}(\nu)}{\cos^2(\alpha_0)}, \tag{4}$$

with incidence mode α_0. According to Ref. [3], an excited mode $\hat{\alpha}_m = \alpha_{\nu=0}$ scatters such that

$$k_{y\nu} = k_{y0} + 2\pi\frac{\nu}{s} \tag{5}$$

with the propagating scattered mode order $\nu = \pm1, \pm2, \ldots$. The incident geometrical mode m and the scattered geometrical mode m' are therefore coupled by the scattered mode order ν:

$$m' = m + B\nu. \tag{6}$$

This is equivalent to the Tyler-Sofrin rule [4] and therefore the incidence mode order m can be interpreted as the azimuthal mode order m of a 3D acoustic duct mode.

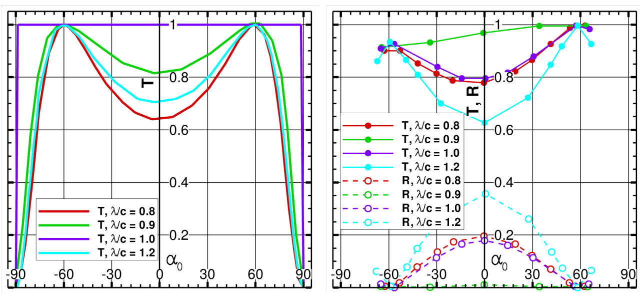
Thus a sufficient method is found to simulate the sound transmission through a 2D blade row for discrete incidence angles without flow.

Results

The simulations show that the condition (5) for the scattered modes is sufficient. Also the transition between sub- and super-resonance described by Eq. (4) is found in the simulations.

Transmission and Reflexion

Sub-resonance The evolution of the power transmission factor with regard to the incidence angle is now investigated (see Fig. 3). The theoretical results presented in Ref. [2] are tried to be reproduced. The condition $s << c$ is not really fulfilled in the simulations since $s = 0.2c$. Due to this difference, some discrepancies are obtained compared to the theory.



(a) Analytical results [2]   (b) Numerical results

Figure 3: Comparison for the transmission factor T and reflection factor R over the incidence α_0 ($s = 0.2c$, $\beta = 60°$).

Super-resonance Figure 4 shows some results for sound transmission at super-resonance. At negative incidence angles α_0 most of the energy is reflected back depending on the pitch s.

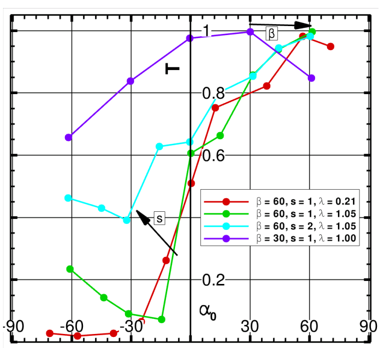
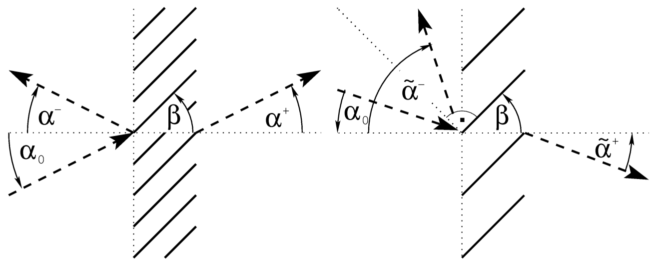


Figure 4: Numerical results for the transmission factor T over the incidence α_0.

Scattering

In general, the energy of the incidence mode scatters in case of super-resonance into all propagating modes. While Kaji and Okazaki did not predict quantitatively the scattering, our simulations show a main direction of energy transport.

For sub-resonant waves the cascade works as an impedance mismatch with reflection perpendicular to the cascade plane (see Fig. 5(a)), whereas super-resonant waves tend to be reflected perpendicular to the blade (Fig. 5(b)) in agreement with the ray theory [5].



(a) Sub-resonance: reflection perpendicular to cascade direction   (b) Super-resonance: most energetic modes are reflected perpendicular to the blade

Figure 5: Main direction of transmission and reflection.

Conclusions

A CAA methodology with periodic boundary conditions was set-up in order to simulate the sound transmission through a blade row without flow. Even though the propagating angles $\hat{\alpha}_m$ are discrete and imposed by the geometrical conditions, they are sufficient to fully describe the scattered modes α_ν.

The classification into sub- and super-resonant modes is also found in the CAA simulations. The predicted sub-resonant behaviour slightly differs from the analytical description [2] due to the non infinite blade spacing that has to be considered in the CAA simulations. The super-resonant state is successfully simulated and a main direction of the most energetic modes could be identified.

References

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