Sound Source Terms of Coaxial Jets

Seong Ryong Koh1, Wolfgang Schr¨oder1, Matthias Meinke1
1 Institute of Aerodynamics RWTH Aachen University, 52062 Aachen, Germany

Introduction

Jet noise is one of the significant phenomena for the design of aircraft engines. Since 1952 a vast number of publications appeared on sound generation due to turbulence interacting with shear layers, Mach wave radiation, and convecting vortex packets [1]. Lighthill’s acoustic analogy indeed succeeded in predicting the general characteristics of the sound radiation. However, lots of research revealed that more sensitive tools are necessary to determine the definite noise source since a small change of the flow can affect the acoustic field.

When hot jets are considered, the jet temperature, i.e., the so-called entropy contribution, is another significant parameter. Bodony and Lele [2] analyzed turbulent high-speed jets to examine the use of Lighthill’s stress tensor in which a significant phase cancellation between the momentum and entropy components was found. The shear stress and enthalpy flux perturbations which have been manipulated in the acoustic modelings are highly intricate in multiple shear layer flows [3, 4]. In other words, to understand the details of the sound generation process, it requires a systematic investigation concerning the acoustic sources based on a robust and yet highly accurate sound prediction tool.

To compute the acoustic field of hot coaxial jets a hybrid large-eddy simulation/computational aeroacoustics (LES/CAA) approach is applied. That is, a two-step method using large-eddy simulation for the flow field and acoustic perturbation equations (APE) [5] for the acoustic field is used. The source terms in the APE formulation are related to certain noise generation mechanisms and thus, it is possible to analyze the acoustic sources in great detail. Using the noise source terms of the acoustic perturbation equations for a compressible fluid the source strength inside hot coaxial jets is analyzed by discrete Fourier transform. The findings will evidence the acoustic radiation to be intensified by the pronounced temperature gradient.

Governing Equations and Computational Scheme

The governing equations of the flow field are the unsteady compressible Navier-Stokes equations being filtered using the Favre-averaging procedure. The system of equations is closed by an implicit eddy diffusivity approach.

The equations describing the sound propagation are the acoustic perturbation equations (APE). Since a compressible flow problem is tackled the APE-4 system is used [5]. Incorporating the entropy gradient terms and using the linear approximation of the second law of thermodynamics the APE-4 system reads

\[
\frac{\partial \rho'}{\partial t} + \bar{\nabla} \cdot (\bar{\rho}' \bar{u}') = \sigma^2 q_c, \tag{1}
\]

\[
\frac{\partial \bar{u}'}{\partial t} + \nabla \cdot (\bar{\rho} \bar{u}') + \nabla \left( \frac{p'}{\rho} \right) = \bar{\mathbf{q}}_m, \tag{2}
\]

where the right-hand side terms are

\[
q_c = -\nabla \cdot (\rho' \bar{u}') + \frac{\bar{p}}{c_p} \frac{D s'}{D t}, \tag{3}
\]

\[
\bar{\mathbf{q}}_m = -(\omega \times \bar{u})' + T' \nabla \bar{s} - s' \nabla \bar{T} - \left( \nabla \frac{(u')^2}{2} \right). \tag{4}
\]

The first step of the hybrid method is based on a large-eddy simulation for the turbulent jet flow to provide the data of the noise source terms. Then, the corresponding acoustic field is computed by solving the acoustic perturbation equations. The details of the general set-up of the LES method are given in Meinke et al. [6]. The numerical method for the acoustic simulations requires a high spatial resolution in the wave number space and a high temporal accuracy in the frequency domain. To accurately resolve the acoustic wave propagation, the seven-point stencil dispersion-relation preserving (DRP) scheme [7] is used for the spatial discretization and an alternating 5-6 stage low-dispersion and low-dissipation Runge-Kutta method for the temporal integration [8]. A detailed description of the two-step method and the discretization of the Navier-Stokes equations and the acoustic perturbation equations is given in Schröder et al. [9] in a general context.

Flow Configuration

The Reynolds number of the round coaxial jets is \( Re_D = 400,000 \). It is based on the mean velocity of the secondary jet \( U_s \), the jet diameter \( D \), the density \( \rho \), and the viscosity \( \mu \) at the nozzle exit. During the large-eddy simulation a mean velocity profile \( \bar{u} \) using the hyperbolic tangent function [10] is imposed on the shear layer at the nozzle exit. The numerical mesh at this region is shown in Fig. 1 where \( R_p \) is the primary jet radius, \( R_s \) is the secondary jet radius of \( D/2 \). The density distribution at the nozzle exit is modeled by the Crocco-Buseman relation and the ideal gas relation [11]. The Mach number of the secondary stream is \( U_s/a_{\infty} = 0.9 \) and the temperature ratio of the secondary flow and the ambient fluid is \( T_s/T_{\infty} = 1.0 \). The ratio of secondary and primary flow diameter is also fixed as \( D_s = 2D_p \). The coaxial jets possess different temperatures in the primary jet. The cold configuration \( (c_j) \) has a temperature \( T_p = T_{\infty} \), whereas the hot configuration has a primary jet temperature \( T_p \approx 2.7T_{\infty} \). The Mach number of the primary jet
is 1.0 for the cold coaxial jet and for the two hot coaxial jets the Mach numbers are 0.6 ($c_{jh_1}$) and 0.88 ($c_{jh_2}$). The vortex sound will be the major noise source in the cold configuration. When Analyzing the hot coaxial jets another source which is excited by the pronounced temperature gradient between the primary and secondary jets will come into play.

**Result**

The comparison in Fig. 2 emphasizes the impact of the temperature excited entropy sources on the OASPL. Compared with the findings of the single and the cold coaxial jet the axial profile of the hot coaxial jet shows an approximately 5dB higher acoustic pressure. Considering the small difference of 1dB to 2dB between the single and the cold coaxial jet, it can be conjectured that the hot primary jet excites another major sound source yielding a more powerful acoustic energy over a wide frequency range. This second major source is caused by the heat content of the hot primary jet. Note, when the acoustic field of the hot coaxial jet is determined only by the vortex sound source ($L'$) its overall acoustic directivity almost coincides with that of the cold coaxial jet. In other words, the vortex sound source is hardly impacted by the entropy source which makes the difference between the hot coaxial jet and the cold coaxial jet.

**Conclusion**

The flow field and the acoustic field of coaxial jet configurations have been analyzed using the LES/APE hybrid approach. The pronounced temperature gradient within the hot coaxial jets enhances the sound radiation. The Lamb vector perturbation is hardly impacted by the inhomogeneous density field. Furthermore, it is found that the acoustic field of the hot coaxial jets can be accurately determined only when the entropy contributions are considered.

**References**


