# Iterative Determination of the Sound Transmission Loss of Rectangular Thin Plates

Yohko Aoki, Waldemar Maysenhölder
*Fraunhofer-Institut für Bauphysik, 70569 Stuttgart, e-Mail: yohko.aoki@ibp.fraunhofer.de*

Iterative schemes and domain decomposition are attractive methods for solving large-scale computational systems. With regard to sound transmission problems it appears as a natural choice to perform separate calculations in each fluid or solid domain and connect them in an iterative manner. However, it is known that a straightforward realization of this procedure may diverge [1]. Two possibilities to overcome this problem are investigated in this paper. In the first one the handling of the conditions at the fluid-solid interfaces is modified [1]. It is expected that the iteration converges by using suitable linear combinations of the original conditions. The other technique is based on the splitting method with an auxiliary matrix, which can also be used to avoid computationally expensive matrix inversions. The difficulty of the methods lies in the optimal choice of the parameters.

## Naive iteration scheme

The structure considered in this study is a rectangular thin aluminum panel, clamped into a rigid baffle. When a plane sound wave is incident on the panel, the velocity of the panel is approximately given in the modal domain as:

$$\dot{\mathbf{w}}_s = \mathbf{Z}_s^{-1}\mathbf{F}_{\mathbf{BP}} \tag{1}$$

where $\mathbf{F}_{\mathbf{BP}}$ denotes the modal amplitude of the blocked pressure integrated over the surface of the panel, and $\mathbf{Z}_\mathbf{s}$ is the structural impedance introduced in [2]. The model used in Eq. (1) is referred to as blocked-pressure estimate. The vibration of the structure produces a sound pressure field, which is predicted as:

$$\mathbf{F}_{rad} = \mathbf{Z}_f \dot{\mathbf{w}}_s \tag{2}$$

where $\mathbf{Z}_f$ denotes the radiation impedance of the panel, whose analytical formulation is detailed in [2]. In order to highlight the fluid loading effect, a heavier fluid is used in this paper.

For obtaining the exact solution of the structural vibration, the effects of the radiated sound pressure on the structure must be taken into account. An iteration scheme is used to predict the response of the uncoupled solid and fluid domains in sequence, and to exchange the information between the two domains. The velocity of the panel at the *k*-th iteration step is expressed by:

$$\dot{\mathbf{w}}_s^{(k+1)} = \mathbf{Z}_s^{-1}\left[\mathbf{F}_{BP} - \mathbf{Z}_f\,\dot{\mathbf{w}}_s^{(k)}\right] \tag{3}$$

The blocked-pressure estimate can be used as starting point for the iteration process $\dot{\mathbf{w}}_s^{(1)} = \mathbf{Z}_s^{-1}\mathbf{F}_{BP}$. This iteration converges, if the spectral radius $\rho$ of the matrix $\mathbf{Z}_s^{-1}\mathbf{Z}_f$ is smaller than unity. $\rho$ of a matrix $\mathbf{B}$ is given by:

$$\rho(\mathbf{B}) \equiv \max|\lambda| \ . \tag{4}$$

where $\lambda$ denotes an eigenvalue of matrix $\mathbf{B}$.

Figure 1 compares the exact sound transmission loss (black line) with the blocked pressure estimate (blue solid line) and the iteration scheme at $k$=50 (red dotted line). The background of the figure is colored in yellow at frequencies where $\rho < 1$. This figure shows no convergence at frequencies near the natural frequencies of the plate. The reason is related to the fact that the impedance matrix $\mathbf{Z}_\mathbf{s}$, describing the response of the plate in vacuum, is not defined at resonant frequencies without damping.

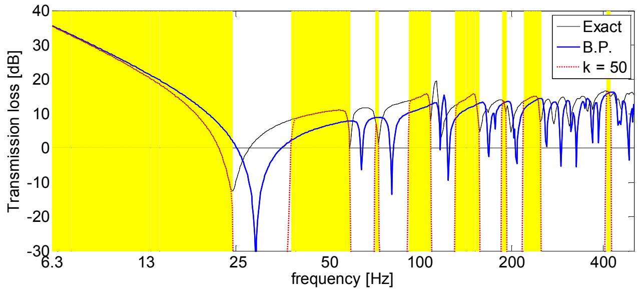


**Figure 1** Sound transmission loss

This tendency is confirmed by Figure 2, in which the row is highlighted by blue or yellow color at frequencies, where $\rho < 1$. The top row is computed with low loss factor $\eta$=0.005, while the bottom has high loss factor $\eta$=0.5. As the loss factor increases, the convergence frequency range is improved. With $\eta$=0.5, the iteration scheme converges for the complete frequency range considered in this simulation.

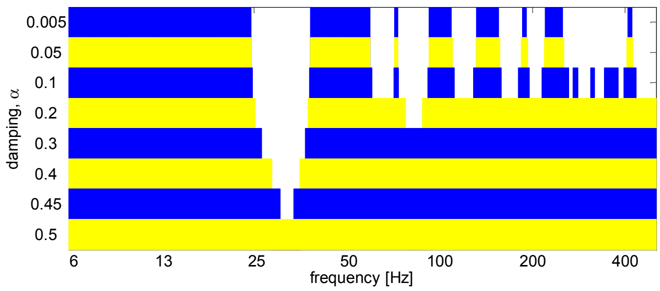


**Figure 2** Convergence range with various loss factors.

## Modified iteration schemes

In this chapter we introduce two methods to improve the convergent frequency range of the iteration scheme.

(1) Modified handling of interface conditions
As the standard approach leads to divergent iteration around the natural frequencies of the panel, the original interface conditions are replaced by suitable linear combinations [1]. The new interface conditions are addressed as:

$$\begin{aligned} &\mathbf{F}_{rad} + \alpha\mathbf{F}_{BP} - \alpha\mathbf{F}_{rad} = \mathbf{Z}_s\dot{\mathbf{w}}_s + \alpha\mathbf{F}_s \\ &\mathbf{F}_s + \beta\,\mathbf{Z}_f\dot{\mathbf{w}}_s = \mathbf{F}_{BP} - \mathbf{F}_{rad} + \beta\,\mathbf{F}_{rad} \end{aligned} \tag{5}$$

where $\mathbf{F}_\mathbf{s}$ denotes the reaction force on the incident sides of the panel, and $\alpha$ and $\beta$ are referred to as relaxation parameters, which can be scalar or matrix. For simplification, $\alpha = \beta$, and they are set to be scalar. Using Eq. (5), the velocity of the panel at the *k*-th iteration step is expressed by:

$$\dot{\mathbf{w}}_s^{(k+1)} = [\alpha\,\mathbf{A} + \mathbf{I}\,]\,\mathbf{F}_{BP} - \mathbf{A}\big[\mathbf{Z}_f + \alpha\,\mathbf{Z}_s\big]\,\dot{\mathbf{w}}_s^{(k)}$$
$$\text{where } \mathbf{A} = \big[\mathbf{Z}_s + \alpha\,\mathbf{Z}_f\big]^{-1}. \qquad (6)$$

Figure 3 shows the spectral radius $\rho$ derived using Eq. (4) with $\mathbf{B}$ = $\mathbf{A}$ [$\mathbf{Z}_f + \alpha\mathbf{Z}_s$] at 10 Hz (blue) and near resonances at 27 and 79 Hz (red and black dotted). The magnified view of the blue line (right-hand-side) indicates that the convergence rate can be improved using proper relaxation parameter. However, if $\rho$ of the naive iteration is above unity, the spectral radius of the modified method remains also above unity. As a conclusion, the convergence of the iteration is barely improved by modifying the conditions at the fluid-solid interfaces with identical scalar relaxation parameters.

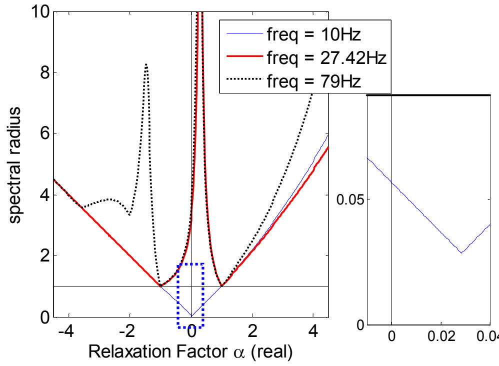


**Figure 3**: Spectral radius with relaxation matrix

(2) Splitting Method
Figure 2 has shown that increasing the plate damping reduces the frequency bands with divergent behavior. Hence the introduction of additional damping, which is compensated for somewhere else in the scheme, could be a promising strategy. After an auxiliary matrix $\mathbf{Z}_A$ is added on both sides of Eq. (3), the iterative formula of the velocity is rewritten as:

$$\dot{\mathbf{w}}_s^{(k+1)} = [\mathbf{Z}_s + \mathbf{Z}_A]^{-1}\big[\mathbf{F}_{BP} - \big(\mathbf{Z}_f - \mathbf{Z}_A\big)\,\dot{\mathbf{w}}_s^{(k)}\,\big]. \qquad (7)$$

The problem now lies in the choice of the free parameters of $\mathbf{Z}_A$, which should assure quick convergence as well as a reduction of numerical effort. In order to reduce the numerical load of the iteration scheme, the off-diagonal entries of the auxiliary matrix are defined by the off-diagonal entries of the structural impedance matrix with opposite sign, such that [$\mathbf{Z}_s$+$\mathbf{Z}_A$] becomes a diagonal matrix. Therefore, the simulation load to calculate the inverse of [$\mathbf{Z}_s$+$\mathbf{Z}_A$] is reduced to *N* scalar inversions, and thus the simulation time is significantly reduced. The diagonal entries of the auxiliary matrix can be freely chosen. As discussed in the previous chapter, introducing an additional damping seems to be a promising strategy. In this paper identical diagonal entries are applied with the radiation impedance of the infinite panel multiplied by the surface area of the rectangular panel,

$$\left|Z_{diag}\right| = \frac{2\rho_1\,c_1}{\cos\Theta_I}\,l_x\,l_y\,, \qquad (8)$$

where $\rho_1$ and $c_1$ are density and sound speed of air respectively, $\Theta_I$ is the incident polar angle, and $l_x$ and $l_y$ are the dimensions of the panel.

Figure 4 compares the spectral radius with auxiliary matrix (black thick line) and without (blue thin line) using Eq. (4) with $\mathbf{B}$ = [$\mathbf{Z}_s$ + $\mathbf{Z}_A$]$^{-1}$ [$\mathbf{Z}_f$ - $\mathbf{Z}_A$]. With the auxiliary matrix the spectral radius is reduced slightly below unity (see the magnified figure). As a conclusion, the combination of the off-diagonal term of the structural impedance and the diagonal entries with the radiation impedance of the infinite panel brings two advantages: first, convergence is achieved, and second the numerical effort is significantly reduced by designing the modified structural impedance matrix as diagonal.

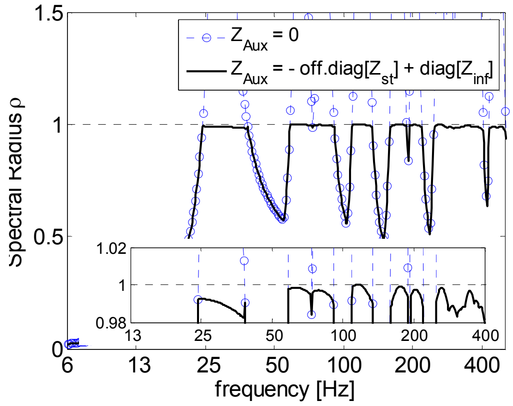


**Figure 4:** Spectral radius with auxiliary matrix

## Conclusions

This paper presents iterative methods to calculate the sound transmission through a thin plate, and the convergence of the iteration schemes has been analyzed. As the standard approach often diverges, we examined two methods for improving the convergence: (1) modified handling of the interface conditions, and (2) splitting. With the first method, a faster convergence rate is observed. However, the convergent frequency range is hardly widened. The splitting method is then studied using an auxiliary matrix, which is defined from the structural impedance and the radiation impedance of the infinite panel. The simulation results indicate that both convergence and simulation load can be significantly improved.

## Acknowledgement

The authors gratefully acknowledge the financial support by the German Research Foundation (DFG).

## References


[1] P. Cummings, X. Feng: Domain decomposition methods for a system of coupled acoustic and elastic Helmholz equations, Proc. of the 11th international conference on domain decomposition methods (1999) 206-213.

[2] R. Woodcock, J. Nicolas: A generalized model for predicting the sound transmission properties of orthotropic plates with general boundary conditions, *J. Acoust. Soc. Am.,* 97 (2) (1995) 1099–1112.