Transient Material Optimization of a Physical Multi-Layered Vocal Fold Model

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Introduction

Understanding human voice generation is a challenging and multi disciplinary task. As the use of in-vivo experiments is very limited, a physical vocal fold model has to be manufactored to study phonation more closely. In this work, we provide a method to retrieve material parameters for a multi-layered model, which resembles given reference movements as close as possible. These reference displacements can either be generated out of in-vivo experiments, e.g. high speed endoscopic images, or obtained using excised larynges.

Numerical Model

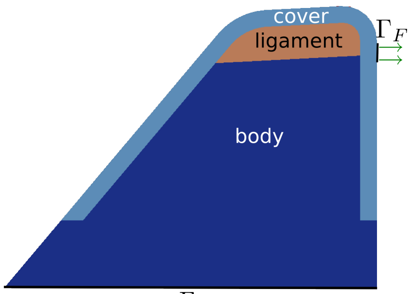

Figure 1: The geometry used for the numerical model of the vocal fold.

In our numerical model, we use a simplified geometry as depicted in figure 1. The dynamic deformation is simulated using the partial differential equation of linear elastodynamics:
Let $\Omega \subset \mathbb{R}^d$ be an elastic body with Lipschitz boundary Γ, that is fixed on part of the boundary $\Gamma_0 \subset \Gamma$ and has a surface force f acting on another part of the boundary $\Gamma_F \subseteq \Gamma \setminus \Gamma_0$. Let $I := [0;T]$ be an interval of time. Find displacement vector field u, such that

$$\begin{array}{rcll} \rho\ddot{u} - \mathrm{div}(\sigma) &=& 0 & \text{in } \Omega \times I \\ \sigma \cdot n &=& f & \text{on } \Gamma_F \times I \\ u &=& 0 & \text{on } \Gamma_0 \times I \\ \sigma &=& C_\alpha \cdot e(u) & \text{in } \Omega \times I \\ \rho &=& \rho_0 \cdot Tr(C) & \text{in } \Omega \times I \\ u &=& u_0 & \text{in } \Omega \text{ for } t = 0 \\ \dot{u} = \ddot{u} &=& 0 & \text{in } \Omega \text{ for } t = 0, \end{array} \quad (1)$$

where $C_\alpha \in L^\infty(\Omega)^{d' \times d'}$ denotes the material tensor. It depends on the material parameters $\alpha \in L^\infty(\Omega)^k$, that can in the most general case vary from point to point and can represent any material law. In the following we however use isotropic material, that is constant on each of the three layers. ρ denotes the density, e the small strain tensor and σ the stress tensor in each point. u_0 is the initial displacement, which in turn is the solution of a static linear elastictity boundary value problem:

$$\begin{array}{rcll} -\mathrm{div}(\sigma_0) &=& 0 & \text{in } \Omega \\ \sigma_0 \cdot n &=& f_0 & \text{on } \Gamma_F \\ u_0 &=& 0 & \text{on } \Gamma_0 \\ \sigma_0 &=& C \cdot e(u_0) & \text{in } \Omega, \end{array} \quad (2)$$

where f_0 is the force f at time $t = 0$. Note that problem (2) is the static version of problem (1) for time $t = 0$.

Optimization Problem

Let u_{ref} be a prescribed discplacement field over the time interval I on part of the boundary $\Gamma_T \subseteq \Gamma$ of the elastic body. We denote with $u_{\text{ref},0}$ the displacement field u_{ref} at time $t = 0$. We consider the following *tracking* problem:

$$\begin{array}{lrcl} \min_{\alpha \in \mathcal{A}} & J(\alpha) & := & w_{\text{stat}} \frac{1}{2} \left\| u_0 - u_{\text{ref},0} \right\|^2_{\Gamma_T} \\ & & + & w_{\text{dyn}} \frac{1}{T} \int_0^T \frac{1}{2} \left\| u - u_{\text{ref}} \right\|^2_{\Gamma_T} \\ \text{where} & u \text{ solves (1)} & & \\ \text{and} & u_0 \text{ solves (2)} & & . \end{array} \quad (3)$$

Problem (3) tries to minimize a weighted sum of the difference of the numerical solution of the static problem (2) u_0 to the reference displacement $u_{\text{ref},0}$ at time $t = 0$ and an avarage over time of the difference of the numerical transient solution u to problem (1) to the reference displacement u_{ref}. In the following, we use $w_{\text{stat}} = w_{\text{dyn}} = \frac{1}{2}$.

Validation

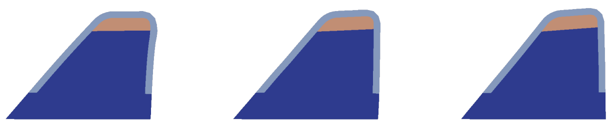

Figure 2: Three timesteps of the deformation used as reference for the validation

To validate our method we run a simulation (figure 2) with given material parameters, extract the displacement at 6 points of the border as the reference displacement field u_{ref}. Then different initial values, two of them are shown in figure 3 and 4, are used to start the optimization from. They result in very different dynamic behavior compared to the reference. The optimization procedure produces out of each initial value different material

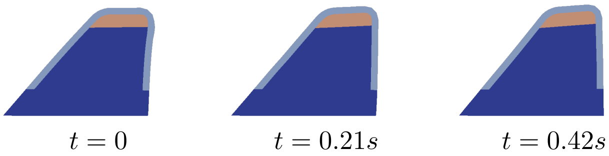

Figure 3: Three timesteps of the deformation generated using initial value with very high Young's moduli

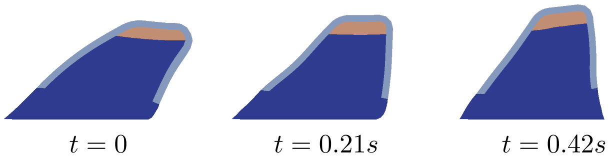

Figure 4: Three timesteps of the deformation generated using initial value with very low Young's moduli

parameters (the two corresponding to the initial values shown in figures 3 and 4 are shown in table 1). All of the resulting displacements are identical to the given reference displacement (avarage distance of the points over time is below $0.01mm$). However as table 1 shows their material parameters differ. As problem (3) is an inverse problem, uniqueness of the solution cannot be expected and this can be observed here as well. Techniques for making the solution unique are adding more load-cases and a regularization approach. Both can easily be integrated in our approach.

Data from Experiments

We use two different experiments for generating the reference displacement. On the one hand we use high-speed endoscopic movies from in-vivo human vocal folds, where a laser projection system is used to generate 3D data and image registration techniques are applied to get displacement fields to be used as reference. On the other hand we use high-speed stereoscopic movies from excised human hemilarynges (figure 5), where sutures are used as markers and their 3D position can be reconstructed. The displacements in these experiments is generated by applying a known force at one of the sutures and then recording the vocal fold returning to its undeformed shape from the moment the force is removed.

Table 2 shows the optimal material parameters for one hemilarynx experiment, so a physical model built using these parameters would dynamically behave as close as possible to the excised vocal fold used in the experiment. The three layered geometry (figure 1) was used

	cover		ligament		body	
	E/kPa	ν	E/kPa	ν	E/kPa	ν
(R)	30.0	0.45	93.0	0.45	55.0	0.45
(1)	34.9	0.49	93.0	0.44	53.9	0.45
(2)	39.2	0.48	85.4	0.45	53.1	0.45

Table 1: The reference material parameters (R) as well as two sets of material parameters from the optimization (1), (2)

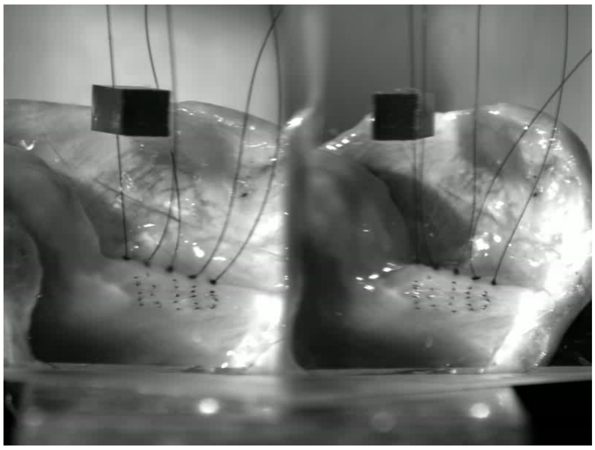

Figure 5: Frame of a stereoscopic high-speed movie of a hemilarynx experiment

	E/kPa	ν
cover	1.35	0.40
ligament	3.60	0.45
body	22.2	0.44

Table 2: Optimal Material parameters from the optimization with reference displacement from the hemilarynx experiments

with isotropic material with constraints $0.40 \leq \nu \leq 0.499$ for the Poisson ratios and $1kPa \leq E \leq 100kPa$ for the Young's moduli of each layer, so that the model can be manufactured using silicone. Figure 6 shows three frames of the corresponding deformation.

Conclusion

In this work we presented a method capable of using in-vitro as well as in-vivo measured data to built a physical multi-layered vocal fold model. Due to the nature of the underlying optimization problem the approach cannot be used for parameter identification of the original material used in the experiment. However, when building the physical model using the optimized material parameters, its dynamic behavior is as close to the original behavior as possible. When measuring material parameters and then approximating these by available material, this approximation does alter the dynamic behavior in an uncontrolled way. In contrast to that, the presented method does consider manufacturing constraints in a way that ensures that the dynamic behavior is approximated optimal for the given material constraints.

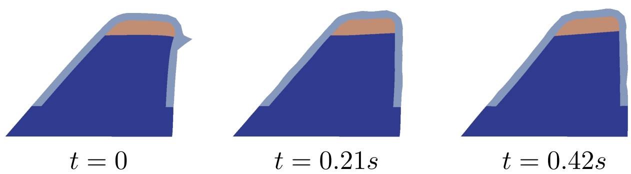

Figure 6: Three timesteps of the deformation generated using material parameters (see table 2) from the optimization with reference displacement from the hemilarynx experiment