Modelling sound transmission through panels using an elemental approach

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Introduction

The elemental approach provides a flexible framework for modelling the structural response and sound radiation from a panel in an infinite baffle. The panel is divided into a grid of elements. The structural response is described by point and transfer mobilities at the element centres. The radiated sound power is calculated from the sum of elemental radiation sources. Both deterministic and stochastic excitation such as an acoustic plane wave and an acoustic diffuse field can be considered. Introducing feedback effects allows modelling the influence of a) fluid loading b) flexible boundary conditions c) passive and active feedback control effects on the panel response and the radiated and transmitted sound power.

The elemental approach

As shown in Fig. 1, the panel is subdivided in a grid of elements. A distributed excitation on the source side is modelled as equivalent point forces. On the receiving side the panel response is sampled at the element centres. For the estimation of the radiated sound power the panel elements are assumed to act as individual piston like radiators [1].

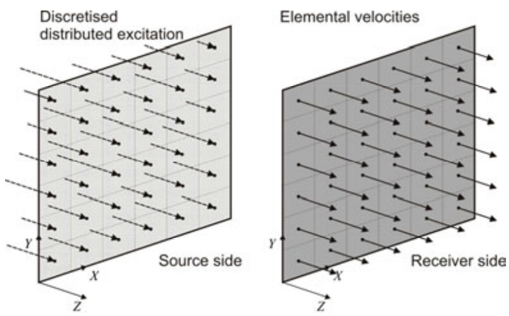

Figure 1: Panel subdivided in a grid of elements.

The grid density is chosen to guarantee at least two elements per shortest wavelength (either that of the disturbance or that of the panel) at the highest frequency of interest. The steady state response is derived assuming time harmonic excitation. For brevity the time harmonic term is omitted in the formulations, which are given in complex form.

Deterministic excitation

For a deterministic excitation the velocity response at the centre of the panel elements is given by

$$\tilde{\dot{\mathbf{w}}}_e = \tilde{\mathbf{Y}}_{ee}\,\tilde{\mathbf{F}}_e\,, \tag{1}$$

where $\tilde{\dot{\mathbf{w}}}_e$ and $\tilde{\mathbf{F}}_e$ are the vectors with the velocities and forces at the centre of the panel elements and $\tilde{\mathbf{Y}}_{ee}$ is the matrix with the mobility functions $Y_{ee,i,j} = \tilde{\dot{w}}_{e,i} / \tilde{F}_{e,j}$.

The total panel kinetic energy and the far field radiated sound power are given by

$$E(\omega) = \frac{m_e}{4}\,\tilde{\dot{\mathbf{w}}}_e^H(\omega)\,\tilde{\dot{\mathbf{w}}}_e(\omega)\,, \tag{2}$$

$$P_{rad}(\omega) = \tilde{\dot{\mathbf{w}}}_e^H(\omega)\,\mathbf{R}_{rad}\,\tilde{\dot{\mathbf{w}}}_e(\omega)\,, \tag{3}$$

where m_e is the mass of a panel element and $\mathbf{R}_{rad}$ is the radiation matrix with the elements

$$R_{rad,ij} = \frac{\omega^2 \rho_0}{4\pi c_0} A_e^2 \left[\frac{\sin\left(k_0\, r_{ij}\right)}{k_0\, r_{ij}}\right], \tag{4}$$

where ρ_0 and c_0 are the mass density and speed of sound in the surrounding media and r is the distance between the element centre coordinates. Note that the formulation in Eq. (3) can also be used to directly determine the radiated sound power from measured structural responses, e.g. from laser vibrometer scans. Also note that the elements of the radiation matrix are proportional to the inverse of the reciprocity relation $\beta/\alpha = (4\pi c_0)/(\omega^2 \rho_0)$, as discussed by Heckel and Rathe [2]. For an acoustic plane wave the elements of the force vector are given by

$$\tilde{F}_{e,n} = 2\,A_e\,\hat{p}_i\,\exp\left(-j\left(k_x x_n + k_y y_n\right)\right), \tag{5}$$

where, $\hat{p}_i$ is the magnitude of the pressure phasor of the incident sound wave, with the wavenumbers $k_x = k_0 \sin(\theta)\cos(\phi)$ and $k_y = k_0 \sin(\theta)\sin(\phi)$, where k_0 is the acoustic wavenumber and θ and ϕ are the incident elevation and azimuthal angles. For an acoustic plane wave excitation the sound transmission coefficient can be calculated from [3]

$$\tau(\theta) = P_{rad} / P_i = P_{rad} / (|\,p_i\,|^2\, A_p \cos(\theta) / 2\rho_0 c_0)\,, \tag{6}$$

Stochastic excitation

For stochastic disturbances the excitation is expressed in terms of the discretised power spectral density

$$\tilde{\mathbf{S}}_{ff} = A_e^2 \Phi \tilde{\mathbf{C}}_{ee}\,, \tag{7}$$

where Φ is the time averaged power spectrum of excitation and $\tilde{\mathbf{C}}_{ee}$ is the spatial cross correlation matrix of the excitation calculated at the element centre locations. For an acoustic diffuse field (ADF), the elements of power spectral density matrix are given by [4]

$$S_{ff,ADF_{ij}}(\omega) = A_e^2\, 4 \left\langle p^2 \right\rangle \sin(k_0\, r_{ij}) / (k_0\, r_{ij})\,, \tag{8}$$

where p is the sound pressure in the diffuse field. Note that power spectral density function has the same spatial characteristics as the radiation matrix in Eq. (4). Also note that other stochastic excitations such as turbulent boundary layer excitation can be described in similar manner [5]. For stochastic disturbances the panel response and sound radiation is given in terms of the spectral densities of the kinetic energy and far field radiated sound power

$$S_E(\omega) = \frac{m_e}{2}\,\mathrm{tr}\left[\tilde{\mathbf{Y}}_{ee}^H \tilde{\mathbf{S}}_{ff} \tilde{\mathbf{Y}}_{ee}\right], \tag{9}$$

$$S_P(\omega) = 2\,\mathrm{tr}\left[\left(\tilde{\mathbf{Y}}_{ee}^H \tilde{\mathbf{S}}_{ff} \tilde{\mathbf{Y}}_{ee}\right)\mathbf{R}_{rad}\right], \tag{10}$$

Figure 3 shows an example for the predicted transmission loss through a thin aluminium panel with all sides pinned, which is excited by a plane acoustic waves with angles of incidence φ=45° and θ=0°,45°,70°, and a diffuse acoustic field.

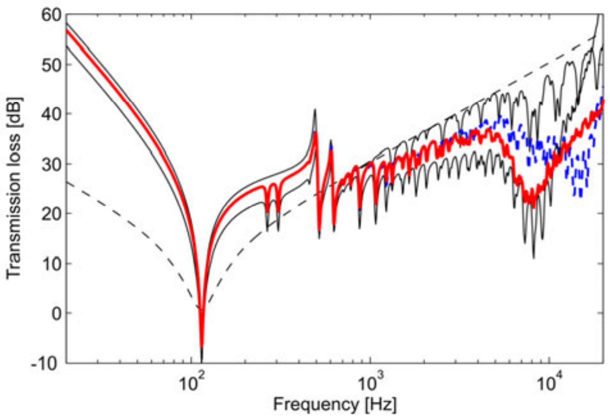

Figure 2: Transmission loss for a plane wave incident at θ=0° and θ=70° (faint), θ=45° (dashed) and for ADF (solid).

Feedback effects

The effects produced by a) fluid loading, b) flexible boundaries, c) active and passive treatments can be modelled as feedback effects. For example, assuming the system is linear, according to the block diagram in Fig. 3, the effect of a discrete velocity feedback forces at control locations c can be included in the model by extending Eq. (1) to

$$\tilde{\mathbf{w}}_e = \tilde{\mathbf{Y}}_{ee}\,\tilde{\mathbf{F}}_e + \tilde{\mathbf{Y}}_{ec}\,\tilde{\mathbf{F}}_c \tag{11}$$

where $\tilde{\mathbf{Y}}_{ec}$ is the matrix with the mobility functions $Y_{ec,ij} = \tilde{w}_{e,i} / \tilde{F}_{c,j}$ and $\tilde{\mathbf{F}}_c$ is the vector with the feedback forces,

$$\tilde{\mathbf{F}}_c = -\tilde{\mathbf{Z}}_c\,\tilde{\mathbf{w}}_c \quad , \tag{12}$$

where $\tilde{\mathbf{Z}}_c$ denotes the impedance of the feedback loop. Substituting Eq. (12) into (11), gives the panel response as

$$\tilde{\mathbf{w}}_e = \left[\tilde{\mathbf{Y}}_{ee} - \tilde{\mathbf{Y}}_{ec}\,\tilde{\mathbf{Z}}_c\left(I_c + \tilde{\mathbf{Y}}_{cc}\,\tilde{\mathbf{Z}}_c\right)^{-1}\tilde{\mathbf{Y}}_{ce}\right]\tilde{\mathbf{F}}_e. \tag{13}$$

Note that the feedback impedance $\tilde{\mathbf{Z}}_c$ can model the effect of various control measures such as lumped masses $\tilde{\mathbf{Z}}_{c,ij} = i\omega m$, skyhook dampers $\tilde{\mathbf{Z}}_{c,ij} = c$, and springs $\tilde{\mathbf{Z}}_{c,ij} = k/(j\omega)$. Also $\tilde{\mathbf{Z}}_c$ can represent more complex systems such as the base impedance of a proof-mass electro-dynamic actuators for the implementation of velocity feedback control loops [1].

As indicated in Fig. 3 the effect of fluid loading can be include in the model by feedback impedances that act on the centres of the panel elements, which are grouped in the impedance matrix $\tilde{\mathbf{Z}}_{rad}$ that contains the complex radiation impedances of the panel elements which are assumed to act as individual pistons. Different fluids on the source and the receiving side can be considered by assigning corresponding impedances. Flexible boundaries can also be included in the model by feedback impedances that act on the on discrete points along the edges and are grouped in an impedance matrix $\tilde{\mathbf{Z}}_b$. When both feedback forces and moments are considered [6], it is possible to model arbitrary flexible and also dissipative boundaries. Note that the accuracy of the model will depend on the accuracy of the expressions for the panel modes, which for freely supported panels are only approximate.

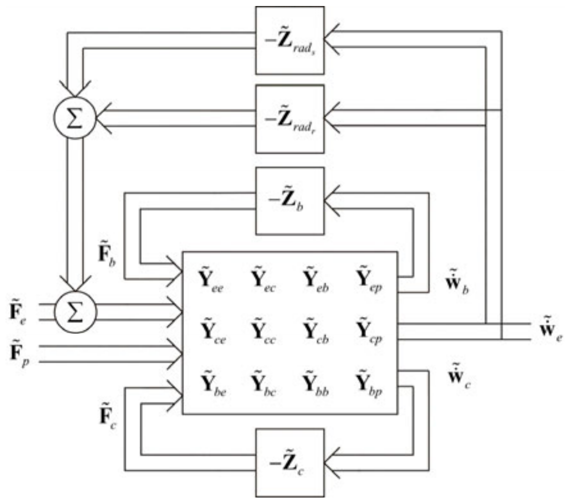

Figure 3: Block diagram of panel model with feedback loops.

Summary

The elemental approach provides a flexible framework for modelling the structural response and sound radiation from panels. Introducing feedback loops allows for systematic studies on the effects of various parameters on the sound transmission. The main drawback of the method is the computational effort which restricts its application to either small scale panels or to a limited frequency range.

Literatur

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