Interface mobilities for structure-borne sound source characterization and the description of the transmission process

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Introduction

In October 2009, the European standard EN 12354-5 has been issued [1]. Based on the characteristic power [2], a characteristic structure-borne sound power level is presented for a characterization of sources of structure-borne sound in buildings. The issuing of the standard can therefore be considered a great step towards establishing a characterization of structure-borne sound sources in the manufacturing industry and acoustic consultancies.

The measurement methods for obtaining the characteristic structure-borne sound power level presented in EN 12354-5 allow for a simplified prediction. The reduction to single power levels, however, limits the physical transparency, thereby making low-noise design and optimization difficult. The characteristic power does offer an insight into the underlying physics, but the spatial representation quickly results in an overwhelming amount of data. Furthermore, the required matrix inversion is likely to result in an amplification of measurement errors [2]. The present paper discusses a possible reformulation of the problem for an improved physical insight and numerical stability by incorporating the interface mobilities [3].

Concept of interface mobilities

Among the approaches for design and optimization, the concept of interface mobilities offers the most general and straightforward physical interpretation of the source and receiver data combined with an enhanced numerical stability and is discussed in more detail below.

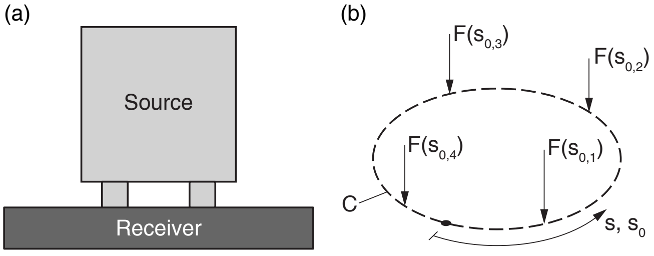

Figure 1: Illustration of multi-point interfaces: (a) multi-point installation; (b) multi-point interface. – – – interface.

For multi-point connections between sources and receivers, a single continuous interface can be formed, which passes all contact points, see Fig. 1. Consequently, the field variables, e.g. forces and velocities, can be treated as continuous and strictly periodic along the interface. By means of a spatial Fourier decomposition, the velocity $v(s)$ can be described in terms of its interface orders $\hat{v}_p$ with the unit [m/s],

$$\hat{v}_p = \frac{1}{C}\int_0^C v(s)e^{-jk_p s}\,\mathrm{d}s, \quad v(s) = \sum_{p=-\infty}^{\infty} \hat{v}_p e^{jk_p s}, \qquad (1)$$

with $k_p = 2p\pi/C$ and $p \in \mathbb{Z}$, where C is the interface circumference. Similarly, the force orders $\hat{F}_q$ are obtained by

$$\hat{F}_q = \frac{1}{C}\int_0^C F(s_0)e^{-jk_q s_0}\,\mathrm{d}s_0, \quad F(s_0) = \sum_{q=-\infty}^{\infty} \hat{F}_q e^{jk_q s_0}, \qquad (2)$$

with $k_q = 2q\pi/C$ and $q \in \mathbb{Z}$, where the spatial force distribution $F(s_0)$ and the force orders are defined to have the unit [N/m].

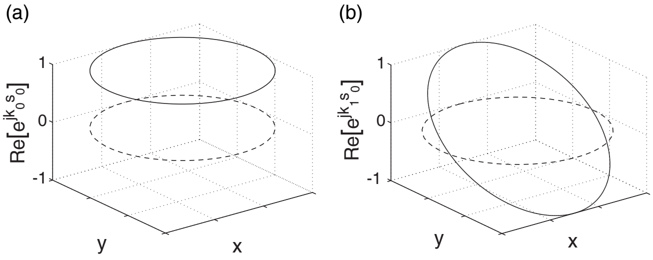

Figure 2: Illustration of interface order distributions along a circular interface: (a) zero order; (b) first order. — interface order; – – – interface.

In Fig. 2, two interface orders are plotted along a circular interface. They could be either force or velocity orders. For the case of the velocity along a circular interface of a source structure, the zero and first orders describe the rigid body motion. The zero-order velocity represents the translational motion, while the first-order velocity describes the rocking motion, see Figs. 2(a) and (b). Velocities of orders larger than one represent the translational component perpendicular to the structure of the vibrational wave field along the interface. When summing up a truncated series of such interface orders with the corresponding complex amplitudes $\hat{v}_p$ or $\hat{F}_q$, the initial spatial velocity or force distribution is approximated, see Eq. (1) or (2), respectively.

By similarly expanding the point and transfer mobilities, the interface mobilities $\hat{Y}_{pq}$ can be written as

$$\begin{aligned}\hat{Y}_{pq} &= \frac{1}{C^2}\int_0^C\int_0^C Y(s|s_0)e^{-jk_p s}e^{-jk_q s_0}\,\mathrm{d}s\,\mathrm{d}s_0,\\ Y(s|s_0) &= \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\hat{Y}_{pq}e^{jk_p s}e^{jk_q s_0}.\end{aligned} \qquad (3)$$

In the general case, a coupling exists between different orders of excitation and response. It has been shown, however, that this cross-order coupling is insignificant with regard to source characterization [3]. The force and velocity orders are therefore connected by the simple equation $\hat{Y}_{p-p} = \hat{v}_p/\hat{F}_p$ and each order can be treated separately. For a four-point installation, there will be the zero and first orders [3] as illsutrated in Fig. 3.

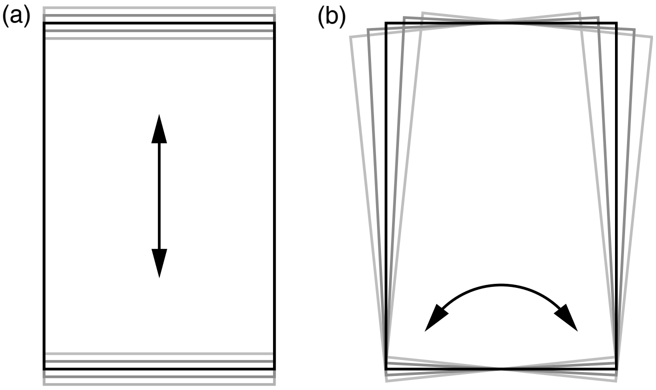

Figure 3: Example of the zero and first order vibration of a source structure: (a) zero order; (b) first order.

Solutions to the integrals

For interfaces with equi-distant spacing between the N contact points, the velocity and force orders and interface mobilities can be rewritten into a discrete Fourier transform of the point and transfer mobilities [3].

$$\begin{aligned}\hat{v}_p &= \frac{1}{N}\sum_{m=1}^{N} v(s_m)e^{-jk_p s_m},\\ \hat{F}_q &= \frac{1}{C}\sum_{n=1}^{N} F(s_{0,n})e^{-jk_q s_{0,n}},\\ \hat{Y}_{pq} &= \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N} Y(s_m|s_{0,n})e^{-jk_p s_m}e^{-jk_q s_{0,n}}.\end{aligned} \qquad (4)$$

The above equations allow the employment of fast Fourier transform algorithms for the calculation of the velocity and force orders as well as the interface mobilities, e.g. `vp=fft(v)/N;` in MATLAB.

Quantities for source characterization

For a characterization of structure-borne sound sources, the source descriptor S and the coupling function C_f were introduced. By incorporating the concept of interface mobilities, these two quantities can be applied for sources with multiple contact points.

$$S_p = \frac{1}{2}\frac{|\hat{v}_{p,FS}|^2}{\hat{Y}^*_{p-p,S}},\ C_{f,p} = \frac{\hat{Y}^*_{p-p,S}\hat{Y}_{p-p,R}}{|\hat{Y}_{p-p,S}+\hat{Y}_{p-p,R}|^2} \qquad (5)$$

The complex power then follows by $Q = \sum S_p C_{f,p}$ and the characteristic power equals $\sum S_p$.

Quantities for the transmission process

In the analysis of structure-borne sound sources, two quantities are of primary interest. Such are the power transmitted to the receiver

$$W = \frac{1}{2}\sum_{p=-\infty}^{\infty}\frac{|\hat{v}_{p,FS}|^2\mathrm{Re}\left[\hat{Y}_{p-p,R}\right]}{|\hat{Y}_{p-p,S}+\hat{Y}_{p-p,R}|^2} \qquad (6)$$

and the vibration amplitude at the contact points

$$v(s) = \sum_{p=-\infty}^{\infty}\hat{v}_{p,FS}\left(1-\frac{\hat{Y}_{p-p,S}}{\hat{Y}_{p-p,S}+\hat{Y}_{p-p,R}}\right)e^{jk_p s}. \qquad (7)$$

Concluding remarks

The interface mobility approach for source characterization allows a reduction of the complex structure-borne sound sources into a few physically comprehensible quantities. For a four-point installation with a single component of motion, these are the zero and first orders as opposed to the 16 different spectra obtained from the characteristic power. In combination with the gained formal simplicity, the interface mobility approach offers physical insight which is essential with respect to low-noise product design. Furthermore, without an inversion of the mobility matrices, the interface mobility approach is found to be numerically more stable. The applicability of fast Fourier transform algorithms for the calculation of the interface mobilities and force and velocity orders facilitates short computation times.

Acknowledgments

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