Multichannel active absorption systems (AAS): Theory and numerical results.

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Introduction

Efficient sound absorption at low frequencies is not always easy to achieve using passive materials like foams or mineral wool, which is due to the large dimensions of materials required and the wavelength involved at that range. In order to find new solutions for this problem, active methods seems to provide a feasible answer. The work presented here introduces a novel mathematical model for a multichannel active sound absorption system based on the wave separation method of Nishimura [1]. The wave separation method is combined with a modal model of a three-dimensional room in order to obtain the controlling parameters of the active system. Theory and simulations are presented. Simulations of the sound pressure field inside a room are presented to show the effect of the active absorption system. For comparison, the presented method is contrasted with classical active noise control strategies.

Modal model of a rectangular enclosure

To describe the sound field within a three-dimensional enclosure the starting point is given by the homogenous wave equation

$$\nabla^2 p(\vec{\mathbf{r}}) + k^2 p(\vec{\mathbf{r}}) = 0 \tag{1}$$

where, as usual, $k = \omega/c$ is the wave number of the excitation signal associated with the angular frequency $\omega = 2\pi f$ and the speed of sound c. Considering the boundary conditions that all the walls that define the enclosure are completely rigid, i.e. that the normal component of the particle velocity at each surface of the enclosure is zero, gives that the general solution of the wave equation (1) is

$$p(\vec{\mathbf{r}}) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} A_{pqr}\Psi_{pqr}(\vec{\mathbf{r}}) \tag{2}$$

where $\vec{\mathbf{r}} = (x, y, z)$ defines any position within the enclosure, A_{pqr} is an arbitrary modal amplitude and $\Psi_{pqr}(\vec{\mathbf{r}})$ is the modal shape function that satisfies the boundary conditions.

The resulting sound field generated by a set of primary p_{prim} and secondary p_{sec} sources located within the rectangular enclosure can be found solving $p(\vec{\mathbf{r}})$ from the equation

$$\nabla^2 p(\vec{\mathbf{r}}) + k^2 p(\vec{\mathbf{r}}) = p_{\mathrm{prim}}(\vec{\mathbf{r}}) + \sum_{i=1}^{I} \beta_i\, p_{\mathrm{sec}}^{(i)}(\vec{\mathbf{r}}). \tag{3}$$

Each secondary source is driven by a *complex coefficient* β_i which is a magnitude-and-phase shift regarding the primary source. For simplicity the right-hand side of equation (3) can be written in a more compact way as

$$= \sum_{i=0}^{I} \beta_i\, p^{(i)}(\vec{\mathbf{r}}) \tag{4}$$

with $\beta_0 = 1$ and $p^{(0)}(\vec{\mathbf{r}}) = p_{\mathrm{prim}}(\vec{\mathbf{r}})$. Considering that all the sources are point sources, the general solution for equation (3) will be given by

$$p(\vec{\mathbf{r}}) = \sum_{i=0}^{I} \beta_i \sum_{pqr}^{\infty} g_{pqr}^{(i)}\, \Psi_{pqr}(\vec{\mathbf{r}}), \tag{5}$$

where

$$g_{pqr}^{(i)} = \frac{p_{pqr}^{(i)}}{1 - \frac{\omega_{pqr}^2}{\omega^2}}. \tag{6}$$

and $p_{pqr}^{(i)}$ are the modal amplitudes generated by the combination of primary and secondary sources positions.

At the beginning of this section, the enclosure was defined as completely rigid, but this condition is not entirely true in real rooms. Hence, to take this into account, it is necessary to include a loss factor η that comprises all the occurring losses. η is usually a small real constant. In this way the new k_{pqr} will be given by

$$k_{pqr}^2 = (1 + j\,\eta)\left[\left(\frac{p\pi}{l_x}\right)^2 + \left(\frac{q\pi}{l_y}\right)^2 + \left(\frac{r\pi}{l_z}\right)^2\right]. \tag{7}$$

where p, q and r are modal indexes for each spatial coordinate and l_x, l_y, l_z are the dimensions of the enclosure.

Control criterion

As proven before [2], it is possible to use the minimization of the sound pressure traveling in one particular direction to generate the active absorption. The criterion chosen for this purpose is the minimization of the reflected sound pressure p_r, which is generated by the impinging sound over the boundaries of the enclosure.

As every active control system, sensors are required to serve as input for observation. Here, a number of L sensors is used and each sensor is located at the position $\vec{\mathbf{r}}_\ell = (x_\ell, y_\ell, z_\ell)$, where $\ell = 1, 2, 3, \ldots, L$. Therefore, the cost function that minimizes the sum of the reflected sound pressure at all L sensor positions is defined as

$$J_{x_p} = \sum_{\ell=1}^{L} \left|p_r(\vec{\mathbf{r}}_\ell)\right|^2 = \sum_{\ell=1}^{L} \left|\frac{1}{2}\Big[p(\vec{\mathbf{r}}_\ell) - \varrho\, c\, u_x(\vec{\mathbf{r}}_\ell)\Big]\right|^2 \tag{8}$$

where the subscript x_p in J_{x_p} denotes the minimization of the reflected component of the pressure p_r generated by

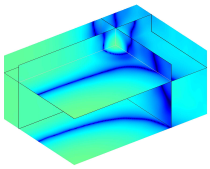
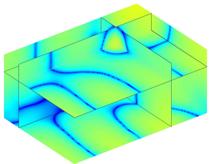
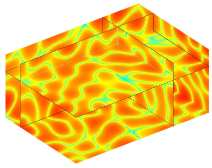
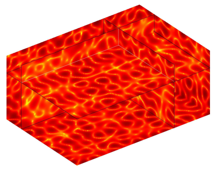
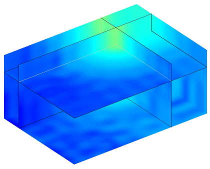
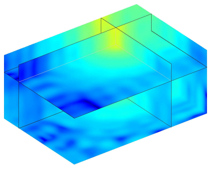
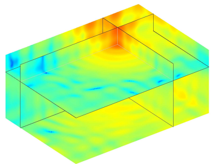
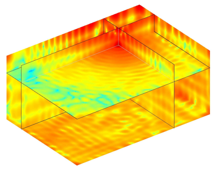
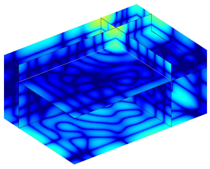
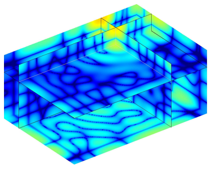
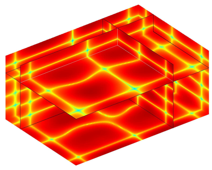
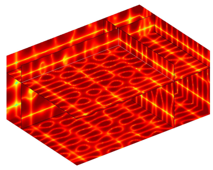
	50 Hz	100 Hz	350 Hz	700 Hz
OFF				
AAS				
ANC				

Table 1: Sound pressure field calculated using three different conditions: control off, active absorption on and active noise control on. Simulations made for a room of dimensions $l_x = 5.5$ m, $l_y = 3.8$ m and $l_z = 2.4$ m.

a wave propagating in the positive direction of x. Solving the cost function using the *Hermitian quadratic form* [3], it is possible to find the optimal set of coefficients $\boldsymbol{\beta}_{\text{opt}}$ that minimizes J_{x_p}. However, in order to achieve total absorption in a particular dimension, let say x, it is essential to take into account also the reflected component p_r due to a wave traveling in the negative direction of x. Hence, the cost function J_x that generates the total absorption in the x dimension will be composed by $J_x = J_{x_p} + J_{x_n}$. The difference between J_{x_p} and J_{x_n} is given by the opposite direction of propagation of p_r.

Finally, to find the cost function that minimizes the reflected pressure for the two other dimensions, i.e. y and z, it is necessary to repeat the procedures described above. The only difference is that now the particle velocities u_y and u_z must be used. Thus, four new cost functions are obtained: J_{y_p}, J_{y_n}, J_{z_p} and J_{z_n}. In the best case scenario, one would want to place secondary sources in each of the six walls of the enclosure to obtain the best possible absorption. Under this conditions the total cost function J that minimizes the reflected pressure for the six directions of propagation inside the room will be given by

$$J = J_{x_p} + J_{x_n} + J_{y_p} + J_{y_n} + J_{z_p} + J_{z_n}. \qquad (9)$$

Later on, the method described here is contrasted with a classical method of active noise control (ANC), where the objective is to locally minimize the sound pressure p at a given number of sensor points $\vec{\mathbf{r}_\ell}$. The cost function for this criterion can be written as

$$J_{\text{ANC}} = \sum_{\ell=1}^{L} |\, p(\vec{\mathbf{r}_\ell})\,|^2 \,. \qquad (10)$$

Numerical results

The first row of Table (1) shows the sound field when only the primary source is present inside the room. There, it is possible to observe the modes distribution within the enclosure with well defined maxima and minima of pressure. In the second row, when the active absorption system is turned on, the maxima and minima of pressure appear to be damped. But the most important characteristic is that the pressure decreases with the distance with respect to the source in the same way as it would happens with a source radiating in a free field. This behavior is the most clear evidence that the active system is effectively absorbing sound and is able to cancel almost completely the reflection coming from the walls. The third row shows the sound field obtained when the pressure is locally minimized at every sensor position. Here, as in the figures of the first row, it is possible to observe distinct patterns of maxima and minima of pressure. This means that some modes are damped but other modes are amplified and a shift in the spatial distribution of the sound field has occurred.

Concluding remarks

A mathematical model for the active sound absorption was presented. This model is the result of a combination between the modal model of a room and the minimization of the reflected part of the sound field. The simulations show that it is possible to obtain almost a free field condition inside a rigid enclosure, proving that the active system is effectively absorbing.

References

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[3] P.A. Nelson and S.J. Elliott, *"Active Control of Sound"*, Academic Press, 1992.