Active Vibration Control of Piezoelectric Structures

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Introduction

Mechanical lightweight structures, used in mechanical and civil engineering applications, often tend to unwanted vibrations, which may result in disturbing sound radiation or even damage of components. Passive methods for increasing the structural damping are often inadequate, because they always include the use of additional mass in the form of damping materials, additional stiffening or mass dampers. The concept of active vibration control has become a useful approach in the recent years, due to improvement of the vibration susceptibility of lightweight structures with the least possible increase in mass. For the active vibration control, supporting mechanical structure is supplied with sensors and actuators operated by a controller. High integration of the structural system with active materials (actuators/sensors) and control is regarded as a smart structure due to its ability to adapt to environment changes. The technology of smart materials and structures, especially piezoelectric smart structures, has become mature over the last decade. One promising application of piezoelectric smart structures is the control and suppression of unwanted structural vibrations.

In this paper an overall approach to active control of piezoelectric structures will be presented by an example of a smart cantilever beam, which involves subsequent steps of model identification, control design, simulation, experimental verification and implementation.

Experimental Setup

A Flexible smart structure shown in figure 1 is used as an experimental object to test the effectiveness of the proposed vibration suppression method.

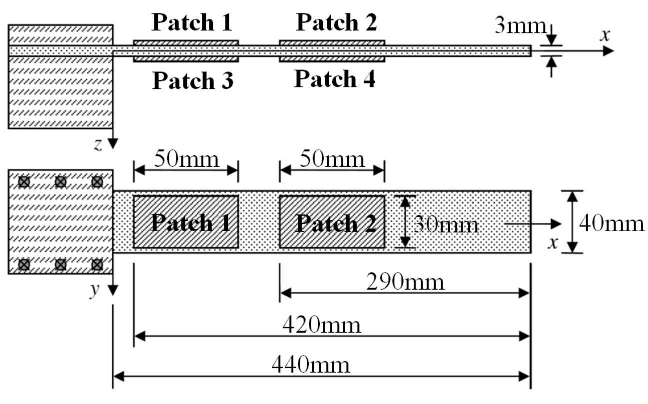

Figure 1: Geometry and layout of the smart structure

The smart structure consists of a cantilever aluminum beam (Young's modulus 70 GPa and density 2.7 g/cm³) and four piezoelectric patches (DuraActTM P-876.A15), which are attached to the beam, two on each side of the beam. These four patches are used as actuators to enable active vibration control of the beam. A scanning digital laser Doppler vibrometer (VH-1000-D), which acts as a sensor, is used to measure the velocity of the bending vibration at a certain point, near the free end of the beam. The sensor provides the feedback signal in the active control algorithm.

In this experiment the plant has four voltage inputs, one input for each piezoelectric actuator, and one output, which is recorded from the sensor. For implementing the controller in real time, a dSPACE digital data acquisition and real-time control system is used. The dSPACE system is connected both to an analog-to-digital converter and to a digital-to-analog converter, in order to process the continuous-time sensor signal and for generating a continuous-time series of control signal, respectively. Since for the actuation of the piezoelectric patches high voltages are required, the control signal is fed out of the DAC board through the piezo-amplifier to the piezoelectric actuators. The control law, for the active suppression of the bending vibration, is designed on MATLAB platform and then downloaded to the dSPACE digital data acquisition and real-time control system to implement the proposed control algorithm.

Subspace-based System Model Identification

Controller design for smart structures relies on accurate modeling of the system dynamics. Therefore in this investigation the subspace approach *n4sid* is used to obtain experimentally a model of the piezoelectric smart structure in the state-space form.

The subspace approach is as an alternative to numerical modeling using the FEM approach, since in case of availability of the real structure it enables a successful modeling of a MIMO system in the state-space representation

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{\Phi x}[k] + \mathbf{\Gamma u}[k] \\ \mathbf{y}[k] &= \mathbf{Cx}[k] + \mathbf{Du}[k] \end{aligned} . \qquad (1)$$

The subspace identification method *n4sid* offers a numerically reliable algorithm for computing a state-space description directly from the measured input/output data. The computations are based on QR-factorization and singular-value decomposition, for which numerically reliable algorithms are available [1].

Since the subspace identification is based on sampled input/output measurement sequences $\{\mathbf{u_k}\}$ and $\{\mathbf{y_k}\}$, the method applies to a discrete-time form of the resulting state-space model, with discrete-time state and control matrices $\mathbf{\Phi}$ and $\mathbf{\Gamma}$, respectively.

Model-based Controller Design

For the purpose of reducing the vibrations of the smart beam, caused by a mechanical disturbance, a negative feedback control loop can be established, where the state variables in the state space representation of the structural model are amplified and fed back to the actuators. Thus, assuming a negative feedback control law proportional to the state variables of the system, the control voltage can be written as

$$\mathbf{u}[k] = -\mathbf{L}\mathbf{x}[k], \tag{2}$$

where $\mathbf{L}$ represents the feedback gain matrix and $\mathbf{x}$ is the state vector of the design model. Substituting (2) into the state equation of the state space representation (1), the closed-loop system state equation can be written as

$$\mathbf{x}[k+1] = (\mathbf{\Phi} - \mathbf{\Gamma}\mathbf{L})\mathbf{x}[k]. \tag{3}$$

The feedback gain matrix $\mathbf{L}$ controls the system response through the modification of the closed-loop system poles. There are a variety of techniques available to design the feedback gain matrix $\mathbf{L}$ [2]. In this paper a linear quadratic regulator (LQR) is used to determine the feedback gain. The controller design task is to determine the control law which minimizes the cost function

$$J = \sum_{k=0}^{\infty}\left(\mathbf{x}[k]^{\mathrm{T}}\mathbf{Q}\mathbf{x}[k] + \mathbf{u}[k]^{\mathrm{T}}\mathbf{R}\mathbf{u}[k]\right), \tag{4}$$

where, the matrices $\mathbf{Q}$ and $\mathbf{R}$ are the designer specified symmetric positive definite weighting matrices.

A subsequent step in the LQR design for smart structures is the estimation of the state variables. The feedback gain control law, as presented in (2), requires the state variables of the design state vector in order to supply the control input. Since the state variables of the identified model do not represent physically realistic and measurable values, a Kalman-filter [2] is used as an optimal observer for the estimation of the state vector, where measurements at the time k are used for obtaining an estimate of the state vector at the time k+1. Since the estimate is predicted one step ahead, it is expressed in the form

$$\hat{\mathbf{x}}[k+1] = \mathbf{\Phi}\hat{\mathbf{x}}[k] + \mathbf{\Gamma}\mathbf{u}[k] + \mathbf{K}\left(\mathbf{y}[k] - \mathbf{C}\hat{\mathbf{x}}[k]\right), \tag{5}$$

where $\hat{\mathbf{x}}$ is an estimate of the state vector $\mathbf{x}$ and $\mathbf{K}$ is the feedback gain matrix of the prediction estimator. The difference between the estimate and the actual value of $\mathbf{x}$ is:

$$\mathbf{e}[k] = \mathbf{x}[k] - \hat{\mathbf{x}}[k]. \tag{6}$$

The Kalman-filter design problem is treated as a weighted recursive least squares estimation problem. The feedback gain matrix $\mathbf{K}$ is based on minimizing estimation errors. The equations for computing the feedback gain $\mathbf{K}$ have a striking resemblance to the equations for computing the optimal LQR gain.

The Kalman-filter implementation requires a priori knowledge of the process noise magnitude $\mathbf{R}_w$ and the measurement noise magnitude $\mathbf{R}_v$. The value for $\mathbf{R}_v$ in the given actual design problem has be chosen based on the sensor accuracy.

Investigation Results

For the purpose of experimental validation, the identified model coupled with the Kalman state estimator and LQ controller designed based on the identified model are implemented within a real time configuration. The closed loop system for the active vibration control of the beam is implemented on the real time data acquisition platform of the dSPACE system with sampling frequency of 1 kHz.

Successful performance of the controlled system is demonstrated for the case of the initial displacement disturbance. Free vibrations of the beam caused by an initial displacement applied to the tip of the beam are comparable with impulse disturbance vibrations.

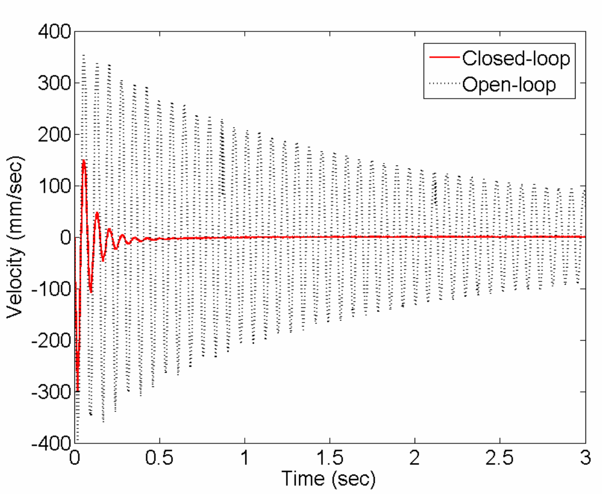

Figure 2: Free vibration response (velocity) of the controlled and uncontrolled system

The free vibration response (velocity) of the open-loop and closed-loop system subjected to an initial displacement of 8mm is measured using the laser vibrometer at the point, which is located 22mm away from the free end, and it is represented in figure 2. Designed controller attenuates significantly the magnitudes of the free end displacement. The closed-loop 5% settling time is equal to 0.3 s, which reveals a great improvement of the response attenuation when compared with the open-loop one (7.9 s).

References

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