Inverse Methods for Estimating Blocked Forces of a Structure-Borne Sound Source on a Reception Plate

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Introduction

The characterization of structure-borne sound sources is the topic of many research projects. Besides a thorough theoretical understanding of the underlying principles, simple measurement methods are needed that can be standardized and applied by non-experts to make predictions about the resulting noise level of the source when installed. A good example is the reception plate method, as described in [1]: the operating source is attached to a known reception plate and the average plate velocity recorded. From this, the total structure-borne source power can be calculated. One disadvantage of this method is that only the sum of the squared contact forces can be determined. This paper introduces an approach based on the reception plate method, which uses inverse methods to quantify individual contact forces.

Inverse Force Determination

For a structure-borne sound source connected to a receiving structure, the contact forces $\mathbf{F_c}$ between source and receiver and the velocities $\mathbf{v_{rem}}$ at remote positions on the structure are linked by the receiver transfer mobility matrix $\mathbf{Y_{rem}}$:

$$\mathbf{v_{rem}} = \mathbf{Y_{rem}}\mathbf{F_c} \quad \text{and} \quad \mathbf{F_c} = \mathbf{Y_{rem}^{-1}}\mathbf{v_{rem}} \qquad (1)$$

To obtain the contact forces, it is therefore sufficient to measure the receiver transfer mobility matrix and the velocities on the structure during operation of the source. Moorhouse et al. [3] use the *coupled* mobilities of source and receiver to calculate the blocked forces of the source. If a high-mobility source situation is assumed ($\mathbf{Y_s} \gg \mathbf{Y_r}$) as in the standard reception plate approach, the procedure can be greatly simplified, as the contact forces approximate the blocked forces ($\mathbf{F_c} \approx \mathbf{F_b}$). If further a well known receiver structure is used such as a thin plate with known boundary conditions, the mobility matrix can be calculated.

An aluminium plate of size 2.12 m × 1.50 m × 0.02 m is used as receiving structure (Figure 1). The plate is supported at the corners and edges by visco-elastic patches, effectively making it a free (FFFF) plate. The point and transfer mobilities for the normal force component can be calculated according to [2]:

$$Y_{v_z F_z}(\omega) = j\omega \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\psi_{mn}(x_2,y_2)\psi_{mn}(x_1,y_1)}{\rho h l_x l_y \left[\omega_{mn}^2(1+j\eta)-\omega^2\right]} \qquad (2)$$

Here, ψ_{mn} are the plate mode shapes, ω_{mn} are the corresponding eigenfrequencies, h, l_x and l_y are the dimensions of the plate, ρ is the density, and η is the loss factor.

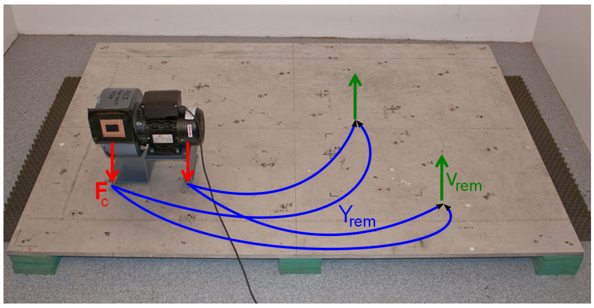

Figure 1: Aluminium reception plate. The plate is supported at the corners and edges by visco-elastic patches.

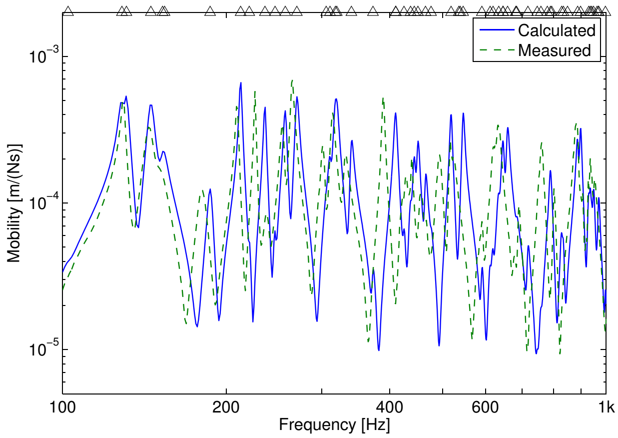

Figure 2: Point mobility of aluminium reception plate.

The plate mode shapes can be calculated from the mode shapes of free beams. The point and transfer mobilities of other components of excitation (forces *and* moments) can be calculated in a similar way [2].

Figure 2 shows the calculated and measured point mobility at an arbitrary position on the plate. The triangles at the top indicate the calculated eigenfrequencies of the plate. Whilst there is agreement in terms of "signature", there is a shift in the resonance frequencies, resulting in large differences for individual frequency components. Since the calculation in Equation 1 is applied for each individual frequency, errors will be introduced by these shifts. For the remainder of this paper, measured mobilities are used to eliminate this source of error.

Equation 1 contains another source of error: Matrix inversion is prone to magnify measurement errors if the matrix is ill-conditioned. A low condition number κ indicates a well-conditioned matrix. Thite [4] investigates various methods to reduce the condition number of a mobility matrix: over-determination, singular value rejec-

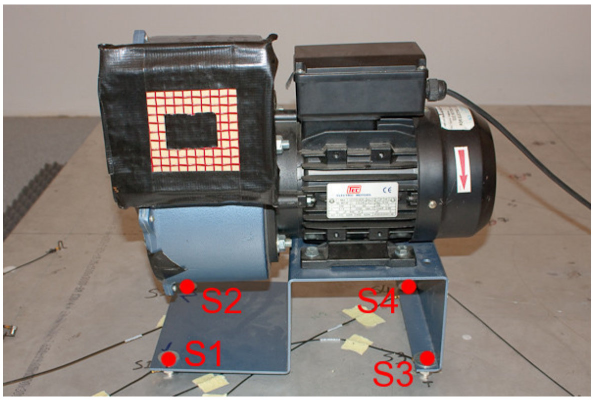

Figure 3: Industrial fan unit used as exemplary source.

tion, Tikhonov regularization, and others. The choice of sensor location also has a significant influence on the matrix condition. One important question is how to know in advance which combinations of velocity response positions are good and which are bad. Thite [4] describes a procedure to quickly identify favorable combinations using the "composite condition number". This procedure has been applied, and results show that while a low composite condition number means almost always a low average condition number, high composite condition numbers do not necessarily mean high average condition numbers.

Results for an Industrial Fan Unit

An industrial fan unit on four feet was considered (Figure 3). The fan unit was mounted on the reception plate via four force transducers (used for comparison). These are considered to be part of the source, thus not changing the contact condition. Forces at the stiff end of the fan base (S3 and S4) were approximately 20 dB higher compared to forces at the resilient end (S1 and S2). The fan unit had a tonal spectrum, with the principal frequency component at 50 Hz.

Figure 4 shows measured and calculated contact forces for the fan unit. Five velocity responses were used (overdetermination by one) and a favorable combination of response positions was chosen. For clarity, only one high force and one low force are shown. The other two forces are comparable. Figure 5 shows the deviations in the force estimates in dB. The high forces are predicted within 5 dB, whilst the low forces are over-predicted. Inverse force determination was performed for many other combinations, and this was a general observation: low forces/moments, in the presence of high forces/moments, will generally be over-predicted.

Conclusions

It could be shown that the proposed modified reception plate method allows identification of dominant components of excitation. Other smaller components are generally over-predicted. A careful choice of velocity response positions is necessary to avoid problems with ill-conditioned mobility matrices. Over-determination and

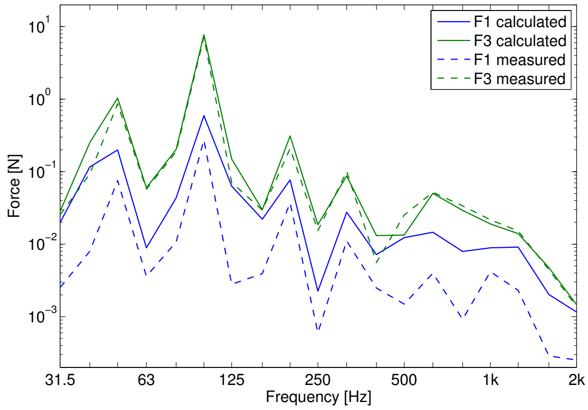

Figure 4: Forces for fan unit on reception plate.

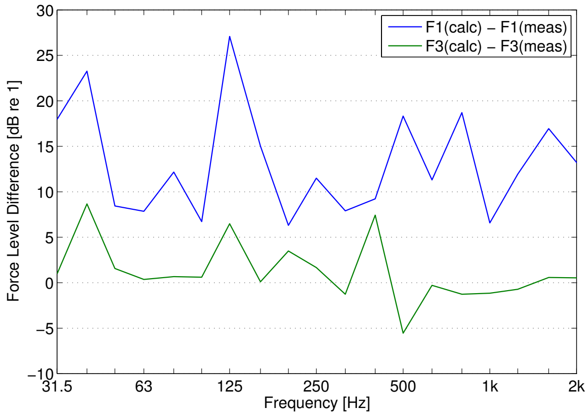

Figure 5: Deviations in force estimates for fan unit.

regularization methods can improve the matrix condition. The unsatisfactory agreement of measured and calculated plate mobilities will be investigated further.

Acknowledgements

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