A hybrid FEM/SEA approach for acoustical FSI-problems
Efficiency studies depending on subsystem definitions and averaging techniques

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Introduction

The prediction of structure borne sound in vehicles or buildings and the related sound fields in the acoustic volume is typically carried out either with a Finite Element (FEM) approach or with the help of energy methods like the Statistical Energy Analysis (SEA). Whereas the first method is limited to lower frequencies the SEA is appropriate for frequency ranges, where the structures have a high modal density and the system´s behavior is dominated by resonant vibrations.

SEA-like Approach

In the mid-frequency range, which is between 200 Hz and 800 Hz in structures of the automotive industry for example, neither the FEM nor the SEA approach is able to provide realistic results. In the scope of this contribution the efficiency of an energy flow analysis based on a "SEA-like" averaging is studied. Here the structure and the fluid are modeled with the help of finite elements compared to a numerical Power Injection Method focusing on coupling and damping loss factors.

The finite element equations for the Fluid-Structure Interaction (FSI) are given by:

$$\begin{bmatrix} \mathbf{M}_s & \mathbf{0} \\ \rho\mathbf{R}^T & \mathbf{M}_f \end{bmatrix}\begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix}+\begin{bmatrix} \mathbf{C}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_f \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{p} \end{bmatrix}+\begin{bmatrix} \mathbf{K}_s & -\mathbf{R} \\ \mathbf{0} & \mathbf{K}_f \end{bmatrix}\begin{bmatrix} u \\ p \end{bmatrix}=\begin{bmatrix} \mathbf{F}_s \\ \mathbf{F}_f \end{bmatrix} \quad (1)$$

where the subscript s denotes the structure and f specifies the fluid. **R** is the coupling matrix between the structure and the fluid.

A system is divided into several subsystems. In contrast to the traditional SEA the requirement of weakly coupled subsystems is not necessary. Using the time-averaged subsystem energies **E** as the state variable, the governing equations are obtained for an averaged ensemble of coupled subsystems in the steady state:

$$\mathbf{E} = \mathbf{AP} \quad (2)$$

Or:

$$\Omega\mathbf{LE} = \mathbf{P} \quad (3)$$

The matrix of Energy Influence Coefficients (EIC) **A**, where A_{ij} describes the energy of subsystem i due to an unit input power in subsystem j, can be solved using the energy matrix **E** and the input power matrix **P**:

$$\mathbf{A} = \mathbf{EP}^{-1} \quad (4)$$

In Eq. (3) **L** defines the matrix of damping and coupling loss factors and **P** the diagonal matrix of input power. Comparing Eq. (3) with Eq. (4), the Loss Factor matrix **L** can easily be calculated by inversion of **A**:

$$\mathbf{L} = \frac{1}{\Omega}\mathbf{A}^{-1} = (\Omega \cdot \mathbf{A})^{-1} \quad (5)$$

If the inversion is allowed, applying the Power Injection Method the input power and the subsystem energy in the steady state are used to predict coupling and damping properties of the system. The subsystems are excited successively. The time-averaged input power of the structural subsystems and the acoustical subsystem is defined by the dot product of the implied force and the resulting velocity at the point of excitation. The time-averaged kinetic and potential energy of the structural and acoustical subsystem are estimated as the sum of the kinetic and potential energies of all elements in the subsystem [2].

Examples

U-Profile: The U-Profile [1] sketched in Fig. 1, which is composed of four plates, was introduced to demonstrate the sensitivity of five averaging techniques listed in Tab. 1.

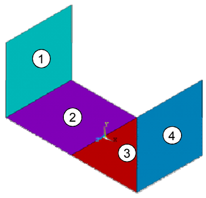

Figure 1: Geometry and substructures

Each plate is defined as an individual subsystem and the above-mentioned SEA-like approach was applied to calculate the damping and coupling loss factors of the subsystems excited by a rain on the roof excitation.

Table 1: Variants of the averaging techniques

	Averaging of the matrices			
Variant	**E ; P**	**A**	**Ω·A**	**L**
1				■ ; ▲
2		■		▲
3		■	▲	
4	■			▲
5	■		▲	
■ Averaging over simulations				
▲ Averaging over frequency of excitation				

Five different averaging techniques were taken into account. The averaging over time was always carried out in the energy matrix and in the input power matrix at the beginning of the post-processing phase. Because the EIC matrix is dependent on the frequency, the averaging over the frequency band is better to be executed after scaling the EIC matrix with the frequency of excitation **Ω**, which means averaging either in matrix **Ω·A** or in matrix **L**.

The subsystems 2 and 3 are strongly coupled, hence the coupling loss factors η_{23} and η_{32} shown in Fig. 2 and 3 are very sensitive in the low frequency averaging intervals, e.g. η_{32} of the variant 1 in the 6^{th} frequency interval is positive, whereas η_{23} is even negative at the same place. In case of the symmetrical system (see Fig. 1) the modal densities of subsystems 2 and 3 are equal. Applying the reciprocity relation $\eta_{ij} \cdot n_i = \eta_{ji} \cdot n_j$, the coupling loss factors η_{23} and η_{32} should be the same. Regarding to this aspect as assessment criterion the second variant was considered as the best averaging process and it will be applied in the next example. The strength of coupling is dependent on the frequency and the results become more stable in the high frequency range.

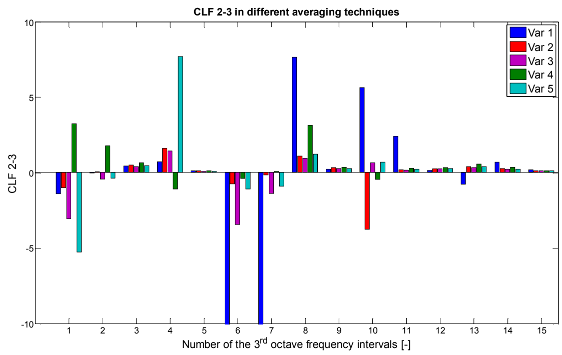

Figure 2: Coupling loss factors η_{23} at subsystems in dependence on the different averaging techniques

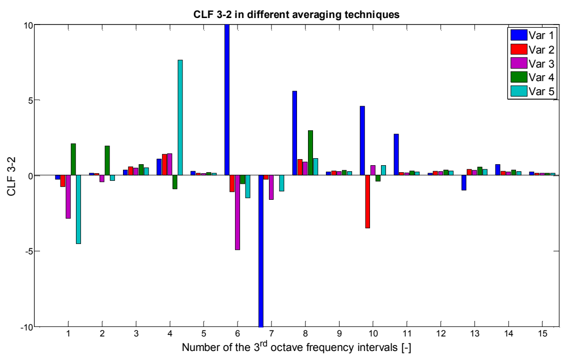

Figure 3: Coupling loss factors η_{32} at subsystems in dependence on the different averaging techniques

Box Model: A box model [1], which consists of 6 plates in the bound and an air volume in the interior, was used to present the influence of diverse subsystem definitions on the loss factors. The air volume was specified as an acoustical subsystem and the plates were defined as the structural subsystems as shown in Fig. 4. Thus, there are in total 2, 4 or 7 subsystems respectively.

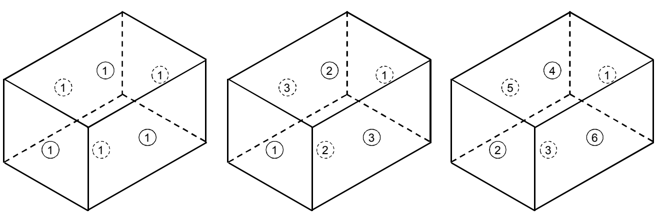

Figure 4: Geometry and subsystem definitions

The loss factors evaluated by the diverse subsystem definitions are exemplarily shown in Fig. 5.

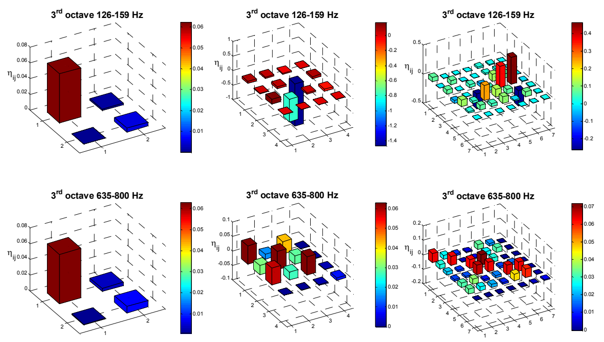

Figure 5: Averaged damping and coupling loss factors

The results in Fig. 5 show that in the low frequency range a refinement of the subsystem definition leads to bad results for the coupling and damping loss factors in case of a strong coupling. As one would expect, for higher frequencies the inversion gives better results.

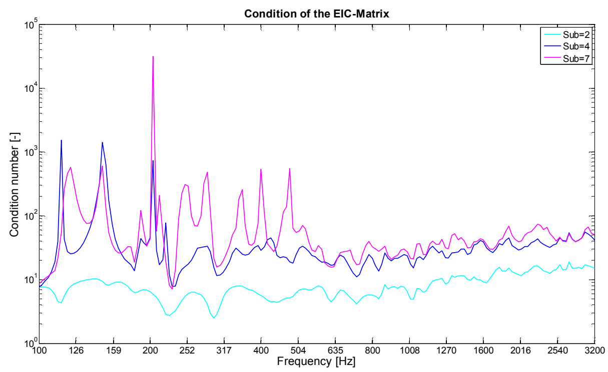

Figure 6: Condition number of the EIC matrix in dependence on the subsystem definition

Fig. 6 illustrates the sensitivity of the EIC matrix and the accuracy of the results calculated by inverting the EIC matrix, which can be described by its condition number. Very often the EIC matrix is getting ill-conditioned for an increasing number of subsystems, especially in the low frequency range.

In the SEA-like approach, which focuses on the EICs, the SEA-requirements as weakly coupled subsystems, which are excited at resonance, and typical simplifications like a rain on the roof excitation are not necessary. The advantages of this method were discussed in this contribution considering the sensitivity of different subsystem definitions. A focus was laid on the comparison of different averaging techniques for the energy-expressions in Tab. 1. The method seems to be promising, especially, if the averaging over the simulations is carried out directly before the inversion of the EIC matrix. In this sense, the second or the third averaging variant is recommended.

References

[1] Xiao, W.: Robuste Modellierung von Strukturen für Anwendungen in der Akustik im mittleren Frequenzbereich mit Hilfe der FEM auf der Basis von Energieflussbetrachtung. Diplomarbeit am Lehrstuhl für Baumechanik der TU München, München, 2010

[2] Mace, B. R. and Shorter, P.J.: Energy Flow Models from Finite Element Analysis. Journal of Sound and Vibration (2000) 233(3), 369-389