Deriving continuous HRTFs from discrete data points

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Introduction

Humans have the remarkable ability to localize the angle of sound incidence with high accuracy. Besides interaural level and time differences, there is also variation in the spectral components of the signal due to diffraction and scattering on the upper part of the human body [1]. These directional dependent head-related transfer functions (HRTFs) are usually obtained as a set of discrete points on a sphere of constant radius around the head. In most applications, these discretized values are used by nearest-neighbor search or interpolation, depending on the required processing speed. Beside the directional dependent ability to localize sound, listeners are also able to distinguish between remote and close sources. For lateral incidence the distance sensitivity is higher, showing audible range dependent differences within a radius of 1.5 m [2]. Spherical harmonic decomposition allows to calculate both range extrapolation and interpolation between discrete angles of radiation, yielding a spatially continuous description of the HRTFs that is expected to enhance the quality in binaural auralization, especially in a dynamic reproduction system where the listener can change the distance to the sound source.

Theory

Duraiswami et al. show in [3] how to use spherical harmonic decomposition for interpolation and extrapolation of HRTFs. A sufficiently dense spherical sampling grid on a fixed radius allows to decompose the HRTFs into a set of spherical harmonic coefficients. This data set can be utilized to gain a value of the HRTFs for any arbitrary direction (interpolation) and to scale the HRTF to different ranges (extrapolation) in order to get a continuous solution for HRTFs at any point outside a confined space containing all significant sources (such as scattering sources).

Using the definition of the spherical harmonics as

$$Y_n^m(\theta_i,\phi_i) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}\cdot P_n^m(\cos\theta_i)\cdot e^{jm\phi_i} \quad (1)$$

with n being order and m degree and $P_n^m(x)$ denoting the associated Legendre function [4]. The index i regards the direction (θ_i,ϕ_i) of the HRTFs.

The interpolation is implicitly performed by finding the coefficients p_{nm} as used in the following equation:

$$p(r,\theta_i,\phi_i,k) = \sum_{n=0}^{+\infty}\sum_{m=-n}^{n} p_{nm}(r,k)Y_n^m(\theta_i,\phi_i) \quad (2)$$

The extrapolation of the HRTFs from the radial distance r_0 to r_1 can then be calculated by scaling the coefficients with an order- and frequency-dependent ratio of spherical Hankel functions:

$$p_{nm}(r_1,k) = p_{nm}(r_0,k)\frac{h_n(kr_1)}{h_n(kr_0)} \quad (3)$$

Inserting this result into Eq. (2) allows to derive spatially continuous function values for the HRTFs at any desired point in space. Hereby a frequency dependent order limit of $n \le N_{\max} = kr_{\min}$ was employed, as suggested in [3] with $r_{\min} = 30\,\mathrm{cm}$ being the radius of a sphere encompassing all scattering sources.

Correlation of spherical functions

To quantify the similarity or dissimilarity of two spherical functions $p_1(\theta,\phi)$ and $p_2(\theta,\phi)$, the correlation can be defined in both the spatial and the spherical harmonic domain [5]

$$C(p_1,p_2) = \oint_{S^2} \overline{p_1(\theta,\phi)}\,p_2(\theta,\phi)\,\mathrm{d}\Omega \quad (4)$$

$$= \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \overline{p_{1,nm}}\,p_{2,nm} \quad (5)$$

with the overbar denoting the conjugate complex. The normalized correlation in the spherical harmonic domain provides a good measure for the similarity of the spherical shape of two functions. Using vector notation $\mathbf{p_k} = \mathrm{vec_N}\{p_{k,nm}\}$ as defined in [6], the normalized correlation can be further expressed as

$$\widetilde{C}(\mathbf{p_1},\mathbf{p_2}) = \frac{\mathbf{p_1}^H\mathbf{p_2}}{|\mathbf{p_1}|\cdot|\mathbf{p_2}|} \quad (6)$$

with $|.| = \|.\|_2$ being the 2-norm of the coefficient vectors [5]. The magnitude of this value will be used to analyze the quality of extrapolation.

Boundary-Element-Method simulation

In order to obtain noise-free data for further comparisons, Boundary-Element-Method (BEM) simulations have been employed to calculate the HRTFs.

HRTFs of a commercially available dummy-head (*HEAD acoustics HMS-III*) were calculated for frequencies from 20 Hz to 6 kHz at 20 Hz steps. The field point grids are equiangular with a resolution of 5° for both elevation and azimuth angle with distances of 30, 50, 100 and 200 cm from the head.

HRTFs for a dummy-head constructed at ITA were calculated for the same radial distances at a frequency range from 20 Hz to 5 kHz with a linear resolution of 20 Hz, and from 5 kHz to the maximum frequency of 16 kHz at 50 Hz steps.

Measurement

The HRTFs for the ITA dummy-head were measured by Lentz [2] for various different radial distances in the semi-anechoic chamber (11 m × 6 m × 5 m) at ITA Aachen. This anechoic room provides a lower cut-off frequency of approx. 100 Hz. A computerized positioning system consisting of a turntable and a rotary arm for a moving sound source allows for measuring HRTFs in the proximal region of the head. Hereby, an equiangular spatial sampling grid with an angular resolution of 5° in elevation and azimuthal direction was applied, which results in 2664 measurement positions on the full sphere. The near-field measurements have been obtained with different settings, compared to the measurement at a radial distance of 2 m. A more detailed description of the measurement setup can be found in [2].

Results and Conclusions

The magnitude of the normalized correlation as defined in Eq. 6 gives a good measure for the similarity of spherical functions. This correlation between simulated/measured and extrapolated HRTFs are plotted in Fig. 1 as functions over frequency.

The solid line describes the quality of outward extrapolation by comparing the extrapolated data obtained at 30 cm with the data obtained at 200 cm. The dashed line rates the quality of inward extrapolation, whereas the dotted line gives the similarity between the original HRTFs of the two different radial ranges and can be considered as deviation of the HRTFs of the two given radial distances. As the close ranges were only measured on a hemisphere and the measurement in a distance of 2 m were obtained with different settings, the compared functions in Fig. 1(c) were of 50 cm and 1 m.

The similarity of extrapolated and simulated HRTF is very high, with a correlation value of nearly one for almost all frequencies. At around 180 Hz the used order truncation causes a local correlation fall-off, as below this frequency the HRTF is modeled as a monopole only. A relaxation of this limit is expected to enhance the result in this low frequency range (cf. [7]).

For the measurement data the improvement of correct extrapolation is much smaller. This is expected to be caused by uncertainties in the measurement procedure. High quality HRTF data thus seems to be essential to employ precise calculation of continuous HRTFs, which generally is a feasible task using the methods given in this paper.

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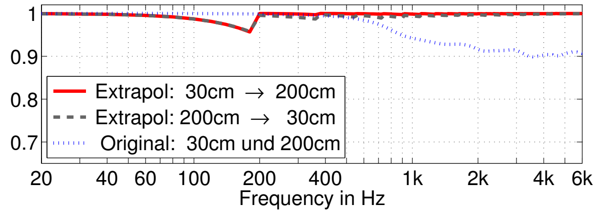
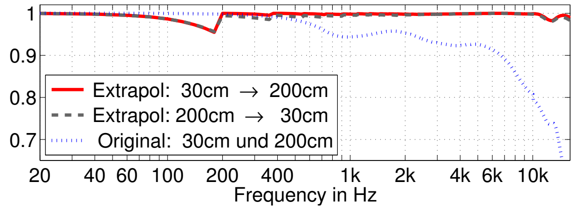
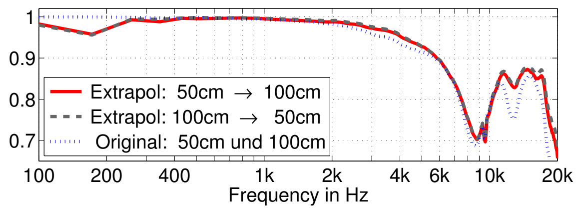


(a) BEM simulation of the HEAD acoustics dummy-head



(b) BEM simulation of the ITA dummy-head



(c) Measurement of the ITA dummy-head

Figure 1: Correlation plots showing the similarity of the extrapolated results with the appropriate original HRTF (outward and inward direction) and the similarity of original HRTFs at different ranges.

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