Room Impulse Responses for Variable Source Radiation Patterns – Part 1: Synthesis

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Introduction

Room impulse responses are generally measured with omni-directional sources lacking information on the behavior of the room for directional sources. Hence, methods were developed to drive the single loudspeaker chassis of compact spherical loudspeaker arrays with individual signals in order to directly approximate certain radiation patterns of target sound sources [3, 4, 6]. This directly implies to measure room impulse responses with approximated radiation patterns, e.g. of a speaker, an instrument, even though only a technical source is present during the measurements.

We propose a novel measurement and synthesis method that measures universal sets of room impulse responses and uses them to synthesize arbitrary radiation patterns after the measurement has been completed. The synthesis method is based on a description of radiation patterns in the spherical harmonic domain.

Method

The proposed method can be divided into two parts, measurement and synthesis, that can also be entirely separated from each other. During the measurement the array is positioned in M diverse orientations using a computerized device. For each orientation the room impulse responses of all N loudspeaker chassis are measured. This leads to a set of $L = M \cdot N$ impulse responses $h(t)$ or its frequency representation $\underline{H}(\omega)$

$$h_l(t) \quad \text{or} \quad \underline{H}_l(\omega) \quad \text{with} \quad l = 1 \ldots L. \tag{1}$$

Each response corresponds to a different radiation pattern. Figure 1 illustrates the method schematically for an array of three loudspeakers: The impulse responses of each driver are measured in two orientations and superposed subsequently which adds up to an extended array of six drivers.

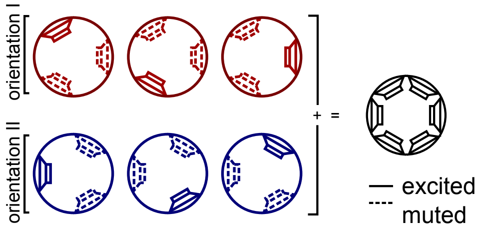

Figure 1: Superposition of several, differently oriented loudspeakers chassis.

Synthesis of Target Responses

In order to approximate the target radiation pattern by the spherical loudspeaker array, complex and frequency dependent weighting factors w_l are determined to obtain the room impulse response $h_\mathrm{T}(t)$ or the transfer function $\underline{H}_\mathrm{T}(\omega)$ of the approximated target radiation pattern by superposition.

$$\underline{H}_\mathrm{T}(\omega) \approx \sum_{l=1}^{L} \underline{H}_l(\omega) \cdot w_l. \tag{2}$$

The superposition approach is only applicable if the room can be considered as a LTI system. Linearity is in general not problematic for air-borne sound paths in the room for moderate sound pressures, whereas time-variances become problematic if the room changes significantly during a measurement session. In order to detect such variances a concept is used as described in [7].

The radiation characteristics of an acoustic source can be described by the directivity factor Γ [2]:

$$\Gamma(\theta, \phi) := \frac{p(r_0, \theta, \phi)}{p(r_0, \theta_0, \phi_0)}. \tag{3}$$

It gives the ratio between the pressure p in arbitrary directions (θ, ϕ) and the pressure at a reference direction (θ_0, ϕ_0) obtained on a radial distance r_0 in the far-field. Hereby common spherical coordinates (r, θ, ϕ) are used[1].

As the directivity value can be regarded as a function which only depends on the radiation angle (θ, ϕ) and which is furthermore square-integrable on the sphere, it can be represented by a set of spherical harmonic coefficients $\hat{\Gamma}_{nm}$ as shown by Williams [5]:

$$\Gamma(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{m} \hat{\Gamma}_{nm} \cdot Y_n^m(\theta, \phi) \tag{4}$$

Y_n^m are spherical harmonics, which can be defined as:

$$Y_n^m(\theta, \phi) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} \, P_n^m(\cos\theta) \, e^{im\phi}. \tag{5}$$

The indices n and m denote order and degree of the function $Y_n^m(\theta, \phi)$. $P_n^m(\mu)$ is the associated Legendre function of first kind.

A radiation pattern of a real source has a finite roughness over the surface. Therefore its characterization can be limited to a maximum order N and the spherical harmonic coefficients can be summarized in a single column vector [6]:

$$\hat{\mathbf{\Gamma}} = \mathrm{vec}_\mathrm{N} \left\{ \hat{\Gamma}_{nm} \right\} \tag{6}$$

[1] In general, p and Γ are complex and frequency dependent, but for better readability they are used without subscripts in the following.

where $0 \le n \le N$ and $-n \le m \le n$.

Each of the earlier mentioned L measured impulse responses with the spherical loudspeaker array correspond to a certain source radiation pattern which can be also written in such a vector $\hat{\mathbf{d}}_l$.

Let $\hat{\mathbf{\Gamma}}_T$ be the radiation pattern of the target to be synthesized, we can formulate analog to equation (2):

$$\hat{\mathbf{\Gamma}}_T \approx \sum_{l=1}^{L} \hat{\mathbf{d}}_l \cdot w_l. \qquad (7)$$

The vectors $\hat{\mathbf{d}}_l$ can be summarized in a matrix characterizing the radiation patterns of the entire extended array $\hat{\mathbf{D}} = \left[\hat{\mathbf{d}}_1 \quad \dots \quad \hat{\mathbf{d}}_L\right]$. Hence equation (7) can be extended towards a matrix formulation,

$$\hat{\mathbf{\Gamma}}_T \approx \hat{\mathbf{D}} \cdot \mathbf{w} \quad \text{and} \quad \mathbf{w} = \left[w_1 \dots w_L\right]^T. \qquad (8)$$

leading to an inverse problem, that can be solved by using the Moore-Penrose pseudo-inverse $\hat{\mathbf{D}}^+$ [1]:

$$\mathbf{w} = \left(\hat{\mathbf{D}}^{\mathbf{H}}\,\hat{\mathbf{D}}\right)^{-1} \hat{\mathbf{D}}^{\mathbf{H}} \cdot \hat{\mathbf{\Gamma}}_T = \hat{\mathbf{D}}^+ \cdot \hat{\mathbf{\Gamma}}_T. \qquad (9)$$

In order to suppress the influence of measurement uncertainties in the synthesis result, the pseudo-inverse is regularized by the Tikhonov method [1].

Experimental Results

In order to evaluate the proposed method a comparative measurement was conducted in a small lecturing hall with a mean reverberation time of approx. 1 s at mid frequencies. Two main measurements were conducted in this room: one measurement with a dodecahedron loudspeaker array (Figure 2, left side), mounted on a computerized turntable to adjust its orientation, and another measurement with a loudspeaker of a certain target radiation pattern that was also used as target response for synthesis (right side). The upper image in Figure 3 shows

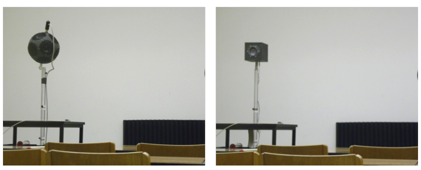

Figure 2: Setup for comparative measurements.

the measurement with the real source and the synthesis result in time domain whereas the lower image zooms into the range of the first reflections in the room impulse response. As can be seen the results look very similar in this representation.

Conclusion

We proposed a measurement method for a special set of room impulse responses and synthesis in a post-processing step for room impulse responses of arbitrary target radiation patterns. The method was validated

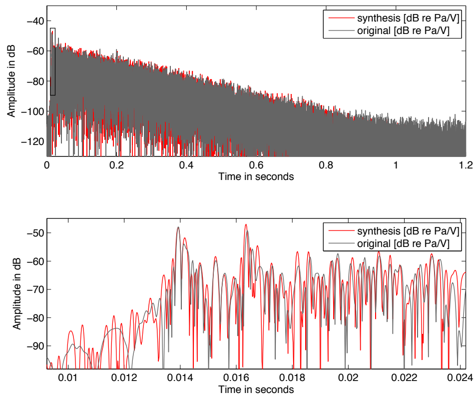

Figure 3: Measured and synthesized impulse response (time domain).

in a small lecturing hall using a 12-channel dodecahedron spherical loudspeaker array with automatically adjustable orientation angles to synthetically increase the number of drivers and therefore the number of different radiation patterns. The results obtained by the synthesis of the proposed method were compared to measurements with the source that was also used as target for the synthesis. The applicability and the limits of the method is discussed in [7].

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