Numerical Simulation of Acoustic Streaming within a Biological Fluid-Structure Coupled System like the Cochlea

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Introduction

In this paper, we present a new approach to simulate acoustic streaming (AS) within a fluid-structure-coupled system. The term "acoustic streaming" refers to the mean motion of a fluid (or a gas) that hosts an acoustic field. Although AS is an old topic, it has not yet succeeded to establish an adequate method for the numerical simulation of AS within biophysical systems like the cochlea. The primary reason for this lies in the fact that the velocity field of the fluid is mainly influenced by the interactions with its adjacent structures. Therefore, we have to pay particular attention to an accurate modeling of this coupling.

This work is motivated by the controversial discussion about the influence of AS within the inner ear (cf. [5]). The results of our numerical simulation yield numerical estimates of the occurence of AS within the inner ear, providing instruments for a discussion on their physiolocical impact.

Methods

The Fluidic System

The system of equations, that we use for describing the dynamics of the fluid within the inner ear, is based on the conservation principles of mass and momentum

$$\frac{\partial \rho}{\partial t} = -\operatorname{div}(\rho \boldsymbol{v}), \tag{1}$$

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\operatorname{grad} \boldsymbol{v}) \cdot \boldsymbol{v}\right) = \operatorname{div} \boldsymbol{\sigma}, \tag{2}$$

where $\boldsymbol{v}$, p, ρ, t and $\boldsymbol{\sigma}$ denote the velocity, pressure, density, time and stress tensor.[1] We must also specify some material-dependent equations. We assume, that the dynamics of the liquid within the inner ear can be modeled as a shear and bulk viscous fluid and furthermore that there is a linear relationship between the stress tensor $\boldsymbol{\sigma}$ and the deformation tensor[2] $\boldsymbol{D}$:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + (\zeta - \frac{2}{3}\eta) \operatorname{tr}(\boldsymbol{D})\boldsymbol{I} + 2\eta\boldsymbol{D}. \tag{3}$$

We also assume a linear dependency between the pressure and the density, given by $p = c_0\rho$, where c_0 is known as the small signal sound speed.

[1] The gradient of a vector is a second-order tensor, defined by $\operatorname{grad} \boldsymbol{v} = \frac{\partial v_i}{\partial x_j}\boldsymbol{e}_i\boldsymbol{e}_j$, where $\boldsymbol{e}_k$ refers to a fixed orthonormal basis.

[2] $D_{ij} := \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$

Perturbation Expansion

This nonlinear fluidic system of equations can be considered as a regular perturbation problem with respect to the small number $\epsilon := \frac{v_0}{c_0} \ll 1$, where v_0 denotes a typical amount of the velocity of the fluid field (cf. [2]). Thus, in order to get a numerical approximation, we expand the flow variables in the usual power series of the number ϵ. For example, the perturbation expansion of the pressure variable is given by

$$p = p^{(0)} + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + \mathcal{O}(\epsilon^3). \tag{4}$$

The superscripts (0), (1) and (2) refer to the ambient, the first order and the second order components of the associated variables. By substituting all flow variables by such a perturbation expansion and by dropping all terms of at least second order, a first order system of equation can be obained, that describes the fundamental acoustic field:

$$\frac{\partial \rho^{(1)}}{\partial t} = -\rho^{(0)} \operatorname{div} \boldsymbol{v}^{(1)}, \tag{5}$$

$$\rho^{(0)} \frac{\partial \boldsymbol{v}^{(1)}}{\partial t} = \operatorname{div} \boldsymbol{\sigma}^{(1)}. \tag{6}$$

Quite similar, the secondary flow can be characterized by a system of equations which consists of the second order terms. Since we are only interested in the secondary steady flow of the fluid motions (and not in the second order harmonic fluctuations), we can apply a mean operator $\langle\cdot\rangle$ which takes the average over a single period. Therefore, a second order system, that describes AS, can be written as

$$\rho^{(0)} \operatorname{div} v^{(dc)} = -\frac{1}{{c_0}^2} \operatorname{div}\langle p^{(1)}\boldsymbol{v}^{(1)}\rangle, \tag{7}$$

$$\operatorname{div} \boldsymbol{\sigma}^{(dc)} = -\frac{1}{{c_0}^2}\langle p^{(1)}\frac{\partial \boldsymbol{v}^{(1)}}{\partial t}\rangle - \rho^{(0)}\langle(\operatorname{grad} \boldsymbol{v}^{(1)}) \cdot \boldsymbol{v}^{(1)}\rangle. \tag{8}$$

The superscript (dc) indicates the time-averaged part of the second order terms. The right hand sides of equations 7 and 8 show, that AS is completely driven by mean values of the acoustic field. The acoustic field, in turn, is mainly characterized by the fluid-structure-interactions between the perilymphs and the cochlear structures, in particular the basilar membrane (BM). Particular attention has to be paid to the boundary condition at the oscillating boundaries of the second order system, in order to guarantee the no-slip condition. Detailed informations about this topic can be found in [1] and [2].

The Cochlear Partition

In order to represent the mechanical characteristics of the BM we adapted a linear model introduced by Mammano et. al. [3]. Mammano proposed to describe the mechanics of the BM by a longitudinal coupled system of damped oscillators

$$m\frac{\partial^2 \xi}{\partial t^2} + h\frac{\partial \xi}{\partial t} + k\xi = f_p, \tag{9}$$

where ξ denotes the vertical displacement of the membrane. The coefficients k, h and m refer to different physical aspects based on measurements and theoretical considerations, that represent the stiffness, the damping and the inertia of the BM. The system is stimulated by the force f_p that is proportional to the difference of the fluid pressure above and below the BM:

$$f_p = w_{bm}\big(p_+^{(1)} - p_-^{(1)}\big). \tag{10}$$

Otherwise, the motions of the fluid at the boundaries of the BM are determined by its displacements, which leads to the boundary condition

$$\boldsymbol{v}^{(1)} = \frac{\partial}{\partial t}\boldsymbol{\xi}. \tag{11}$$

Equations 10 and 11 specify the coupling between the biological structures and the fluid within the chambers. This is only a simplified representation of our more complex model, that also includes the outer-hair-cell motility.

Computational Model

In order to get an approximate solution of the fluid dynamics (including first and second order flows), we developed a two-stage method.

The first stage consists of solving the acoustic system (first order subproblem), which comprises of the equations 5, 6, 9, 10 and 11. By the use of the finite-element-method, this initial-boundary-value-problem can be converted into a discretized time-variant system. We use an implicit integration method and at each time step our approach leads to a monolithic system that accomplishes the fluid-structure-interactions in one step.

On the basis of the results of the first order subproblem, we applied - at a second stage - a finite-element-discretization to Equations 7 and 8. Now, AS can be determined by solving a single stationary system of equations.

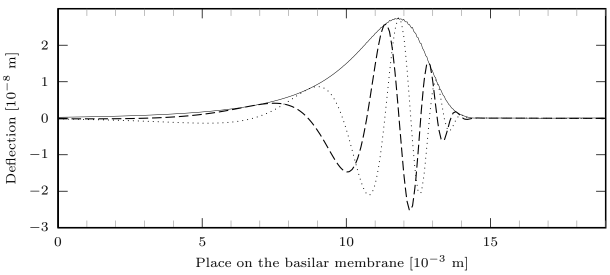

Figure 1: The absolute value (solid line), the real part (dashed line) and the imaginary part (dotted line) of the deflection of the BM evoked by a sinusoidal stimulation of the stapes at 1000Hz.

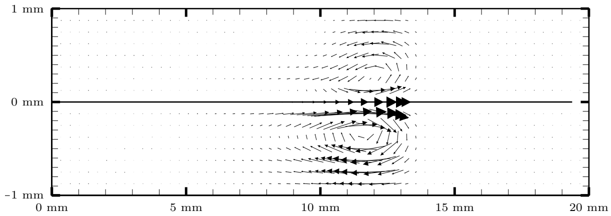

Figure 2: AS computed as solution of the second order system at a frequency of 1000Hz. Near the point of maximal displacement of the BM, two eddies can be observed.

Results

Our computational method was applied to a simplified two-dimensional model, that represents the cochlea of a guinea-pig. Typical traveling wave motions (cf. figure 1) along the BM can be observed by sinusoidal stimulations of the stapes at different frequencies. The characteristic place (point of maximal displacement of the BM) depends on the actual stimulation frequency and its positions correspond to experimental measurements (cf. [6]). Furthermore, the fluid motions near to the BM are in good agreement to theoretical considerations from Lighthill [1], who developed a mathematical model for the investigation of AS within the cochlea.

The results of the second order system provide two eddies near the characteristic place of the BM (cf. figure 2). These eddies are very similar to the eddies, that were observed by Békésy in his experimental studies of a mechanical model (cf. [4]).

This was the first time, that AS within the inner ear has been numerically simulated by a computational model.

References

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