# Electro-acoustic transducers in active feedback control for hearing aids

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### Abstract

While delays in the sub-millisecond range are barely audible under standard operating conditions, they impose critical limitations in a feedback-control setup where they are part of a closed-loop system.

As stated by Blauert et al. [1] over 30 years ago, common loudspeakers and earphones are not necessarily minimumphase systems. It was later published by Buck et al. [2] that balanced-armature receivers were found to exhibit an additional delay not present in some other transducer types.

In the context of active feedback control in hearing aids, the excess phase (i.e. the difference between the total phase and the phase of the corresponding minimum-phase system) is examined, cross-checking measurements with simulations based on two-port models. The results for balanced-armature transducers which are typically used in hearing aids are compared to moving-coil transducers which are typically used in consumer electronics.

## Introduction

In an active feedback control setup such as suggested in [3], delays in the closed loop impose critical limitations on the stability margin. Group delays associated with an invertible minimum-phase system can be compensated for by means of inversion and are thus not critical. If zeros in the right halfplane (RHP) in the continuous Laplace domain or a transportation delay cause non-minimum-phase behavior, an inversion would not be stable or causal respectively. Therefore, knowledge about the minimal phase of the transducers is of great interest in the design of a control loop.

Any linear time-invariant (LTI) system can be represented by a cascade of a minimum-phase system and an all-pass system [1].

$$H(\omega) = |H(\omega)| \cdot e^{j \cdot \varphi(\omega)}$$
  
=  $H_M(\omega) \cdot H_A(\omega)$  (1)  
=  $|H_M(\omega)| \cdot e^{j \cdot (\varphi_M(\omega) + \varphi_A(\omega))}$ 

The excess phase is defined as the phase difference between the entire system and the minimum-phase system and is represented by the phase of the all-pass system  $\varphi_4(\omega)$ .

In the case of a transportation delay (e.g. due to sound propagation), the all-pass system is a linear-phase system, i.e. the excess phase changes linearly with frequency. The resulting group delay is constant over frequency and equals the phase delay. In control theory, such a delay is commonly referred to as dead time.

$$-\frac{d}{d\omega} [\varphi(\omega) - \varphi_{M}(\omega)] = -\frac{d}{d\omega} \varphi_{A}(\omega)$$

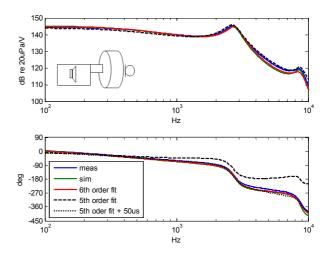
$$\lim_{\omega \to \infty} \frac{\lim_{\omega \to \infty} \varphi_{A}(\omega)}{\omega} = \tau_{dead\_time} \qquad [s] \qquad (2)$$

## **Balanced-armature transducer**

Two-port models for a balanced-armature transducer, tubing and cavity were used to simulate the setup in Fig.1.

**Fig. 1:** Cascaded two-port representation of receiver R, tubing T and cavity E (representing capacitive load impedance).

The resulting frequency response for a purely capacitive load impedance of  $130 \text{ mm}^3$  is in good agreement with the corresponding measurement. While assuming a 5<sup>th</sup> order minimum-phase transfer function indicates a dead time in the order of 50µs, a 6<sup>th</sup> order fit proves to be accurate not only for the magnitude, but also for the phase. The fact that both fits provide a good approximation of the magnitude but differ significantly in the phase response highlights the impossibility of making conclusive statements about a time delay based on band-limited data.



**Fig. 2**: Balanced armature transducer: Measurement (blue), simulation (green), 6<sup>th</sup> order minimum-phase fit (red), and 5<sup>th</sup> order fit indicating 50us (180° at 10kHz) dead time (black).

$$\frac{P(s)}{U(s)} = g \cdot \frac{\prod_{k=1}^{K} \left(\frac{s^2}{\omega_{z,k}} + \frac{s}{\omega_{z,k} \cdot Q_{z,k}} + 1\right) \cdot \prod_{l=1}^{L} \left(\frac{s}{\omega_{z,l}} + 1\right)}{\prod_{m=1}^{M} \left(\frac{s^2}{\omega_{p,m}} + \frac{s}{\omega_{p,m} \cdot Q_{p,m}} + 1\right) \cdot \prod_{n=1}^{N} \left(\frac{s}{\omega_{p,n}} + 1\right)}$$
(3)

For the 5<sup>th</sup> order fit, the M=2 pairs of conjugate-complex poles were placed at 2.7kHz (Q=4) and 8.8kHz (Q=6.3) respectively, the N=1 real pole at 500Hz, the K=1 pair of zeros at 8.8kHz (Q=2.7) and the L=1 real zero at 2kHz ( $2^{nd}$  order high-frequency roll-off). For the 6<sup>th</sup> order fit, the M=2 pairs of poles were placed at 2.8kHz (Q=4.4) and 8.8kHz (Q=5) respectively, the N=2 real poles at 550Hz and 10kHz and the L=1 zero at 8kHz (5<sup>th</sup> order high-frequency roll-off).

It is important to note that the coupler does not produce any steady state sound propagation delay. This is evidenced by the fact that length changes of the coupler do not result in any phase changes, so a uniform pressure distribution in a purely capacitive load can be assumed, see Fig. 3.

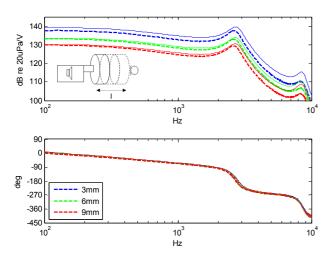
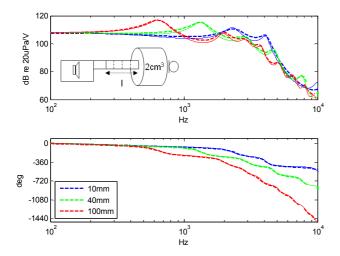


Fig. 3: Simulated (solid) and measured (dashed) frequency responses for a balanced-armature transducer with varying coupler lengths.

In contrast, the sound propagation in the tubing does cause phase changes associated with sound propagation, see Fig. 4.



**Fig. 4:** Simulated (solid) and measured (dashed) frequency responses for a balanced-armature transducer loaded with a cavity of 2cm<sup>3</sup> with varying tubing lengths.

#### **Moving-coil transducer**

For comparison purposes, a moving-coil transducer with 9mm diaphragm diameter loaded with a cavity of 130mm<sup>3</sup> was measured. This transducer type can be approximated by

a  $2^{nd}$  order minimum-phase transfer function, see Fig. 5. For this  $2^{nd}$  order fit, the pair of conjugate-complex poles was placed at 2.6kHz (Q=0.7).

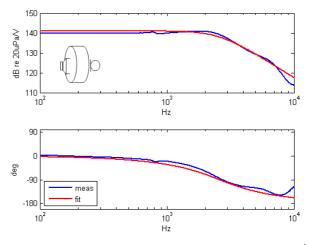


Fig. 5: Moving-coil transducer: Measurement (blue) and 2<sup>nd</sup> order minimum-phase fit (red).

$$\frac{P(s)}{U(s)} = \frac{k}{\frac{s^2}{\omega_p^2} + \frac{s}{\omega_p \cdot Q} + 1}$$
(4)

The advantage of this transducer type in terms of its more convenient phase response is compromised by a larger size, lower high-frequency efficiency, higher sample-to-sample variation and an inferior passive sound attenuation. These aspects render the balanced armature type transducer a better choice for active feedback control in hearing aids.

### Conclusions

No evidence was found for relevant non-minimum phase behavior in neither balanced-armature nor moving-coil transducers. It was shown that the observed phase can indeed be explained by a minimum-phase transfer function within the relevant frequency range for active feedback control. The relative order of the system (degree of high-frequency rolloff) would need to be known for a conclusive statement, but can generally not be determined based on band-limited data.

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#### Literature

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