Radiation modes of t-design and extremal-points compact spherical loudspeaker arrays

Franz Zotter[1], Alexander Mattioli Pasqual[2]

[1] *Institut für Elektronische Musik und Akustik, KU Graz, Email: zotter@iem.at*

[2] *Institut Jean le Rond d'Alembert (UMR 7190), UPMC Univ Paris 06, CNRS*

Introduction

Compact arrangements of independent loudspeakers mounted on a spherical cabinet have been employed as directivity controlled sound sources in research [1, 2, 3, 4]. As opposed to directivity control based on spherical harmonics, the acoustic radiation modes (ARMs) have been recently considered as an alternative basis to control the sound field radiated by compact loudspeaker arrays [5]. Unlike the spherical harmonics, the ARMs always preserve every degree of freedom of the multichannel source. Specifically, regarding the sound power radiated into the far field, these modes are found as eigenvectors of the programmable vibration pattern configuration of the source. Hence, radiation modes can be seen as the natural basis for controlling the vibration patterns of the source to create achievable far field directivities. Associated with each ARM, the eigenvalue indicates its radiation efficiency, which is a useful information for far field magnitude normalization. Nevertheless, regardless of normalization, it would be convenient if the set of ARMs was frequency independent. This contribution discusses the frequency independence for different geometric layouts of spherical arrays. In particular, we focus on t-designs [6] and extremal points for hyperinterpolation [7].

Radiated power

The spherical loudspeaker arrays are modeled here as a spherical surface with L vibrating spherical caps at the locations $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_{\mathrm{L}}$, cf. [1]. The caps have the same size and axisymmetric vibration pattern. Their radial velocities $\boldsymbol{v} = [v_1, \dots, v_{\mathrm{L}}]$ are controlled to produce radiation with variable directivity. The sound power radiated by such arrays can be written in the $\mathrm{L} \times \mathrm{L}$ quadratic form [8]

$$\Pi \propto \boldsymbol{v}^{\mathrm{H}} \, \boldsymbol{B}(\omega) \, \boldsymbol{v}, \tag{1}$$

where the entries of $\boldsymbol{B}$ depend on the wave number k, the array radius a, the vibration pattern of the individual caps, and the angle enclosed by caps l and l'. They are computed by the infinite sum

$$b_{ll'} \propto \frac{1}{k^2} \sum_{n=0}^{\infty} |A_n(ka)|^2 \, P_n(< \boldsymbol{\theta}_l, \boldsymbol{\theta}_{l'} >), \tag{2}$$

where A_n is the axisymmetric *wave spectrum* of a cap (see Ref. [9]), and $P_n(\cdot)$ are the Legendre polynomials.

Radiation Modes

Since $\boldsymbol{B}(\omega)$ is a real symmetric matrix, its eigendecomposition yields a set of real orthogonal eigenvectors for the cap velocities, $\boldsymbol{V}(\omega)$, which corresponds to positive real eigenvalues of the radiated power, $\boldsymbol{\sigma}(\omega)$; i.e.,

$$\boldsymbol{B}(\omega) = \boldsymbol{V}(\omega) \, \mathrm{diag}\{\boldsymbol{\sigma}(\omega)\} \, \boldsymbol{V}(\omega)^{\mathrm{T}}, \tag{3}$$

where the columns of $\boldsymbol{V}(\omega)$ are the ARMs of the source, and the entries of $\boldsymbol{\sigma}(\omega)$ are proportional to the radiation efficiencies associated to the eigenvectors.

A recent paper [8] has demonstrated that the ARMs of the most common array configurations, those of the Platonic solids, do not depend on ω. This raises the question: *Do the ARMs of other layouts get frequency dependent?*

Joint diagonalization. Consider a finite set $\{\boldsymbol{B}_1, \boldsymbol{B}_2, \dots\}$, where $\boldsymbol{B}_i \equiv \boldsymbol{B}(\omega_i)$ and ω_i is a given frequency in the operation range of the array. In general, joint diagonalization of these matrices might not exist, i.e., be only approximate. If it is exact, however, it is equivalent to eigendecomposition at any ω_i.

Simultaneous approximate diagonalization can be obtained by minimizing the off-diagonal terms of $\hat{\boldsymbol{V}} \, \boldsymbol{B}_i \, \hat{\boldsymbol{V}}^{\mathrm{T}}$ for all i. The algorithm in [10] does this using a unitary optimization variable $\hat{\boldsymbol{V}}$ and maximizing the diagonal $\sum_i |\mathrm{diag}\{\hat{\boldsymbol{V}} \, \boldsymbol{B}_i \, \hat{\boldsymbol{V}}^{\mathrm{T}}\}|^2$. The obtained diagonalization is exact if the off-diagonal terms vanish for all i. In this case, eigenvalues and diagonals sorted by magnitude match exactly $\boldsymbol{\sigma}(\omega_i) \equiv \hat{\boldsymbol{\sigma}}(\omega_i) = \mathrm{diag}\{\hat{\boldsymbol{V}} \boldsymbol{B}_i \hat{\boldsymbol{V}}^{\mathrm{T}}\}$.

Clearly, in the case of exact joint diagonalization, the ARMs $\hat{\boldsymbol{V}}$ are frequency independent, as is the case for Platonic arrays. Otherwise, a discrepancy between diagonals $\hat{\boldsymbol{\sigma}}(\omega_i)$ and eigenvalues $\boldsymbol{\sigma}(\omega_i)$ emerges. This can be used to investigate the frequency behavior of the ARMs for alternative spherical array layouts.

Alternative layout examples. Two spherical layout families with strong mathematical features are proposed.

Spherical t-designs [6] are layouts providing the simplest numerical integration rule on the sphere, which is exact for polynomials of degree $l \leq t$. They include the Platonic solids, as indicated in Table 1.

Extremal-points for hyperinterpolation [7] work with the smallest number of sampling points for limited-order interpolation on the sphere. Recently, a 16-driver spherical array based on this layout has been presented [11].

Simulation Results

To study different layouts, 30 log-spaced frequency samples between $0.7 \leq ka \leq 15$ were used for joint diagonalization with an accuracy of 10^{-6}. Table 1 shows the maximum deviation $10 \max_{l,i} |\lg \frac{\hat{\sigma}_l(\omega_i)}{\sigma_l(\omega_i)}|$ [dB] for several t-design and extremal points arrays up to $\mathrm{L} = 36$.

sampling	L	$\max\frac{\sigma}{\hat{\sigma}}$ [dB]
(t=2/extr.) tetrahedron*	4	0.00
(t=3) octahedron*	6	0.00
2-design	7	3.10
(t=3) hexahedron*	8	0.00
2-design	9	1.31
extremal*	9	0.37
3-design*	10	0.00
3-design	11	2.99
(t=5) icosahedron*	12	0.00
3-design	13	9.80
4-design	14	7.48
3-design	15	5.37
5-design	16	1.43
extremal	16	4.21
4-design	17	17.55
5-design	18	8.99
4-design	19	19.04
(t=5) dodecahedron*	20	0.00
4-design	21	9.67
5-design	22	11.98
5-design	23	19.19
3-design	24	7.09
7-design	24	2.99
5-design	25	8.82
extremal	25	13.37
6-design	26	24.82
5-design	27	26.58
6-design	28	11.16
6-design	29	17.40
7-design	30	22.40
6-design	31	24.51
7-design	32	15.61
6-design	33	20.97
7-design	34	14.55
6-design	35	13.58
8-design	36	8.47
extremal	36	18.73

Table 1: Joint diagonal / eigenvalue mismatch for various arrays.

As expected, results reveal that ARMs of Platonic arrays do not depend on frequency. Moreover, the 3-design with L = 10 and the extremal-points array with L = 9 seem to possess approximately frequency independent ARMs. These arrays are marked with (*) in Table 1 and shown in Fig. 1. However, for the remaining layouts, the ARMs may depend significantly on frequency.

Conclusion

We have shown that compact spherical loudspeaker arrays other than Platonic can be expected to have frequency dependent ARMs. Nevertheless, constant ARMs can be assumed for some arrangements with moderate errors, which can be obtained through joint diagonalization. Finally, this is a preliminary numerical study, so that the discussion of practical and efficient ARM-based radiation control is left open for future works.

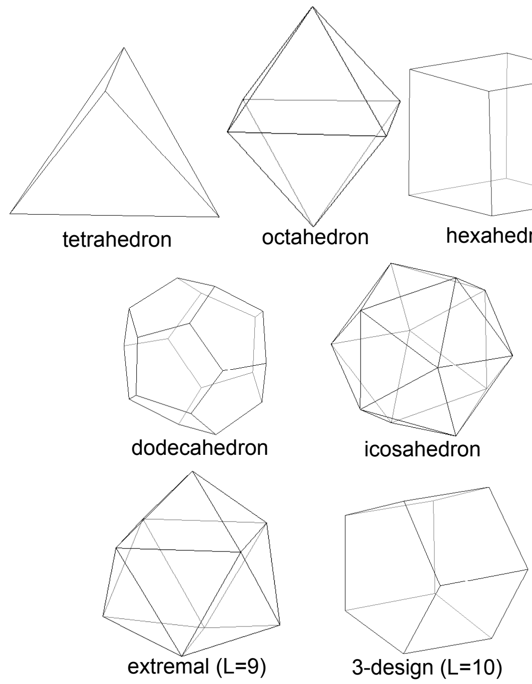

Figure 1: Configurations (*) with completely or nearly frequency independent ARMs.

References

[1] F. Zotter, A. Sontacchi, and R. Höldrich: Modeling a spherical loudspeaker system as multipole source. Fortschritte der Akustik - DAGA 2007, Stuttgart

[2] M. Pollow, G. K. Behler: Variable directivity for Platonic sound sources based on spherical harmonics optimization. Acta Acust. Acust. 95 (2009), 1082–1092

[3] B. Rafaely: Spherical loudspeaker array for local active control of sound. J. Acoust. Soc. Am. 125 (2009), 3006–3017

[4] A. M. Pasqual, P. Herzog, J. R. F. Arruda: Theoretical and experimental analysis of the electromechanical behavior of a compact spherical loudspeaker array for directivity control. J. Acoust. Soc. Am. 128 (2010), 3478–3488

[5] A. M. Pasqual, J. R. F. Arruda, P. Herzog: Application of acoustic radiation modes in the directivity control by a spherical loudspeaker array. Acta Acust. Acust. 96 (2010), 32–42

[6] R. H. Hardin and N. J. A. Sloane: McLaren's improved snub cube and other new spherical designs in three dimensions. Discrete Computational Geometry 15 (1996), 429–441

[7] R. S. Womersley and I. H. Sloan: How good can polynomial interpolation on the sphere be? Adv. Comput. Math. 14 (2001), 195–226

[8] A. M. Pasqual, V. Martin: On the acoustic radiation modes of compact regular polyhedral arrays of independent loudspeakers. J. Acoust. Soc. Am. (under final review)

[9] E. Williams: Fourier Acoustics. Academic Press, 1999

[10] J-F. Cardoso and A. Souloumiac: Jacobi angles for simultaneous diagonalization. SIAM J. Matrix Anal. Appl. 17 (1996), 161–164

[11] M. Kerscher: Compact spherical loudspeaker array for variable sound-radiation. Diploma thesis, University of Music and Performing Arts, Institute of Electronic Music and Acoustics, Graz, 2010