Optimized Measurement System for the Synthesis of Transfer Functions of Variable Sound Source Directivities for Acoustical Measurements

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Introduction

Every sound source features a frequency dependent directivity pattern. This property becomes especially apparent in room acoustical measurements, where the results are inextricably linked with the source directivity [1]. Furthermore, in acoustical simulations, it is desirable to know the room impulse response (RIR) for as many different sound sources as possible.

Conventional Measurement Sources

During previous research, a procedure to synthesize RIRs for sources with an arbitrarily specified directivity was developed [2, 3]. The method superposes single RIR measurements conducted using a multi-channel source with known individual transducer directivities in the spherical harmonics (SH) domain. However, conventional measurement sources such as the dodecahedron used in the preceding studies were shown to not be suitable for the method.

Figure 1 depicts the maximum synthesis error in dB for the dodecahedron. Due to the nature of the synthesis method, it is synonymous with the frequency dependent inversion error energy $E_{\mathrm{E}}(n')$ of the matrix $\mathbf{D}^{[S\times D]}$. q denotes the linear index of the degree m in a SH order n.

$$q = n^2 + n + m + 1 \tag{1}$$

$$\mathbf{R} = \left[\,\hat{\mathbf{r}}_1\,\hat{\mathbf{r}}_2 \cdots \hat{\mathbf{r}}_S\right]^T = \mathbf{D}\cdot\mathbf{D}^{\oplus} - \mathbf{I} \tag{2}$$

$$E_{\mathrm{E}}(n') = \max_{q\forall n=n'} \|\,\hat{\mathbf{r}}_q\|^2 \tag{3}$$

$\mathbf{D}$ contains the S spherical harmonic coefficients of the D individual transducer directivities. $\mathbf{D}^{\oplus}$ constitutes its Tikhonov inversion. $\mathbf{I}$ is an identity matrix. The evaluation of the synthesis error reveals the strong limitations of the dodecahedron system, in regard to the frequency range, as well as the orders.

Optimized Measurement Source

In the current study, a spherical cap model has been used to design a new measurement source. The model provides an approach for the analysis of spherical loudspeakers in the SH domain [4]. It yields the coefficient vector $\hat{\mathbf{a}}$ of an aperture, simulating a membrane of a radius r_{mem} on a sphere with the radius r_{sphere} as a function of the associated aperture angle. The aperture energy $E_{\mathrm{a}}(n')$ is equal to the quadratic sum of the elements of $\hat{\mathbf{a}}$.

$$E_{\mathrm{a}}(n') = \sum_{q\forall n=n'} |\hat{a}(q)|^2 \tag{4}$$

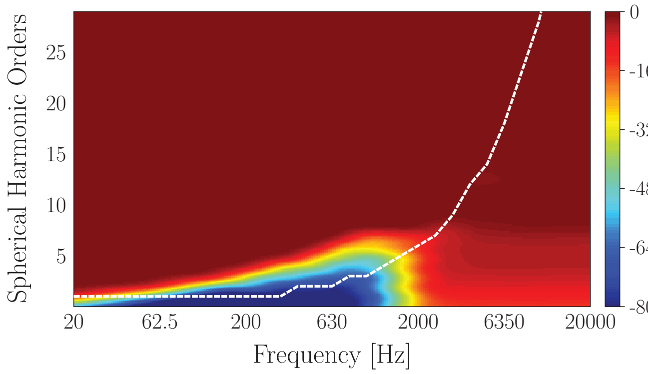


Figure 1: Synthesis error [dB] of the dodecahedron system Red/blue: High/low error. White line: $n = \lfloor k\cdot r\rfloor + 1$

Reducing the aperture angle shifts the energy maximum and zeros towards higher orders, while simultaneously decreasing the absolute amount of energy. Using a carefully selected set of different aperture angles, i.e. different membranes on one sphere, prevents the occurrence of energy zeros in a wide range of orders. Figure 2 depicts the energy distribution of the new measurement source ($r_{\mathrm{mem}} = \{25.4, 38.1, 52.5\}$ mm, $r_{\mathrm{sphere}} = 200$ mm).

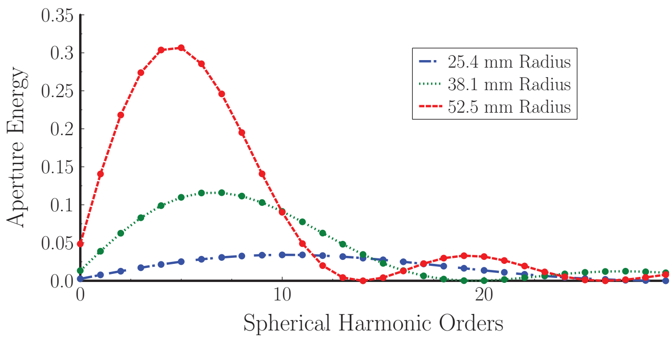


Figure 2: Aperture energy of new measurement system

In spherical harmonics, frequency and order are strongly interconnected [5]. To recreate or record a signal with a maximum frequency $f_{\max}$ correctly, all orders up to $n_{\max}$ have to be regarded, depending on the size of the source.

$$n_{\max} = \lfloor k_{\max}\cdot r_{\mathrm{sphere}}\rfloor = \left\lfloor \frac{2\pi\cdot f_{\max}}{c_0}\cdot r_{\mathrm{sphere}}\right\rfloor \tag{5}$$

A white line in the synthesis error plots in Figure 1 and Figure 4 illustrates the boundary of the respective order ranges.

Converting the apertures $\hat{\mathbf{a}}$ to velocities $\hat{\mathbf{v}}$ introduces the first frequency dependent factor with the wavenumber k, the speed of sound c and the membrane displacement ξ [4].

$$\hat{\mathbf{v}}_{\text{sphere}} = jkc \cdot \xi \cdot \hat{\mathbf{a}} \tag{6}$$

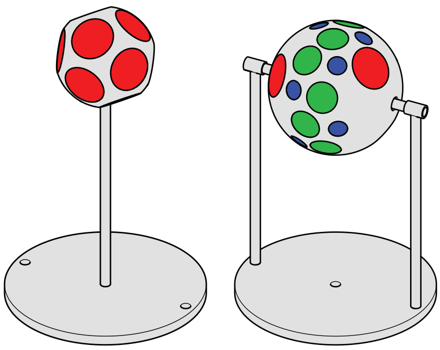
The specific radiation impedance and the far field radiation factor additionally dampen the radiation $\hat{\mathbf{p}}(r_{\text{obs}})$ to a particular observing distance r_{obs}, especially in high orders at low frequencies [6]. h_n herein denotes the Hankel function of the second kind and the n-th order.

$$\hat{\mathbf{p}}(r_{\text{obs}}) = -j\rho_0 c \cdot \frac{h_n(kr_{\text{obs}})}{h_n'(kr_{\text{sphere}})} \cdot \hat{\mathbf{v}}_{\text{sphere}} \tag{7}$$

This effect is inversely related to the sphere radius. Therefore, real sources cannot radiate high orders at low frequencies, which consequently do not have to be synthesized.

The maximum synthesizable order depends on the number and placement of the transducers on the measurement source. Spherical samplings provide strategies for placing points on the confined surface of a sphere. They are geared towards creating a well conditioned base for the spherical harmonics decomposition. However, they differ widely in terms of sampling efficiency.

The Gaussian sampling has an efficiency of $\eta = 0.5$ [7]. Contrary to more efficient samplings, it has points distributed along constant latitudes, allowing for a complete representation through rotation of a single vertical arc of points. Inclining this arc by two optimized angles prior to the rotation doubles the number of sampling points, resulting in a higher sampling order.



(a) Dodecahedron (b) New system

Figure 3: Measurement systems

The new measurement system, shown in Figure 3(b), features vertical arcs of the orders 3 and 11, for the different transducer sizes, distributed over the entire array. Due to construction restrictions, some membranes had to be shifted from their ideal positions. A turntable is used to provide the necessary rotation, a step motor inside the sphere is used to control the inclination.

Results

The simulated synthesis error for the new measurement system is depicted in Figure 4. Recall that the dodecahedron system correctly synthesized RIRs for sources of an order of 4 and a high frequency limit of about 1.5 kHz. The new system is capable of theoretically synthesizing sources up to order 23 at a frequency of 8 kHz, by executing 2688 single measurements.

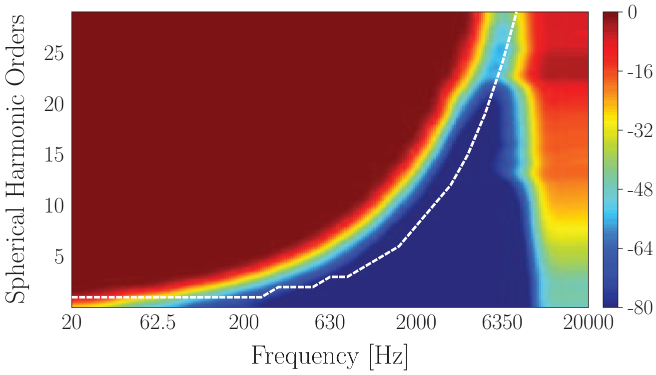


Figure 4: Synthesis error [dB] of the new system. Red/blue: High/low error. White line: $n = \lfloor k \cdot r \rfloor + 1$

Conclusion

The optimized measurement system yields largely improved synthesis results. Due to the dependency of the radiated orders on the size of a source, it is still only possible to fully synthesize RIRs for sources that are equal in size or smaller than the measurement source. Mounting the array eccentrically on the turntable, thus virtually enlarging its outer diameter, would be a countermeasure to this limitation, extending the applicable frequency and order range. However, this would result in longer measurement times, which may violate the assumption of time-invariant rooms.

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