Multi-Model Approach for Computational Aeroacoustics

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Introduction

Since the beginning of computational aeroacoustics (CAA) several numerical methodologies have been proposed. Due to the practical advantages provided by the separate treatment of fluid and acoustic computations, hybrid methodologies still remain the most commonly used approaches for aeroacoustic computations. Concerning the model approach for CAA, we can differ between models based on a perturbation ansatz [1, 2, 8, 6], explicitly taking into account refraction and convection effects, and the inhomogeneous wave equation of Lighthill [5].

Our idea is to apply a multi-model approach, where we solve acoustic perturbation equations for the inner domain (corresponds to the main domain of computational fluid dynamics (CFD)), and for the surrounding domain the convective wave equation of Pierce [7]. Therewith, we can reduce the number of unknowns for the outer domain from four (particle velocity and pressure) to just one (acoustic scalar potential). By applying a Mortar Finite Element (FE) method [3], we can fully include the physical interface conditions between the perturbation equations and Pierce equation.

Formulation

We use the Acoustic Perturbation Equations (APE)[2] for low Mach number flows within the source region, which corresponds also to the CFD region. Therewith, we have to solve the following coupled system of equations

$$\begin{aligned} \frac{\partial p'}{\partial t} + \rho_0 c_0^2 \nabla \cdot \mathbf{u}' + \nabla \cdot (\overline{\mathbf{u}}'\, p') &= c_0^2 q_c\,, && (1) \\ \frac{\partial \mathbf{u}'}{\partial t} + \nabla(\overline{\mathbf{u}} \cdot \mathbf{u}') + \frac{1}{\rho_0} \nabla p' &= \mathbf{q}_m\,. && (2) \end{aligned}$$

In (1) and (2) p' denotes the acoustic pressure, $\mathbf{u}'$ the acoustic particle velocity, c_0 the speed of sound, ρ_0 the mean density of the fluid, $\overline{\mathbf{u}}$ the given mean flow velocity, and q_c, $\mathbf{q}_m$ given acoustic source terms evaluated from CFD computations.

Assuming in the outer region that the source terms are zero and that the flow structures are large compared to the acoustic wave length, we can use the Pierce equation, which reads as follows

$$\rho_0 \frac{D}{Dt}\left(\frac{1}{c_0^2}\frac{D\Phi}{Dt}\right) - \nabla \cdot \rho_0 \nabla \Phi = 0\,; \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \overline{\mathbf{u}} \cdot \nabla\,. \quad (3)$$

In (3) Φ is the scalar acoustic potential, which is related to the acoustic particle velocity $\mathbf{u}'$ and acoustic pressure p' by

$$\mathbf{u}' = -\nabla \cdot \Phi\,; \quad p' = \rho_0 \frac{D\Phi}{Dt}\,. \quad (4)$$

As display in Fig. 1, we will solve for APE in Ω_{a1} and for Pierce equation in Ω_{a2}. Therewith, we have to guar-

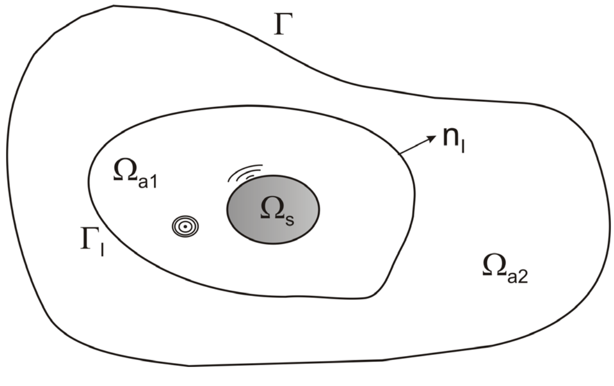


Figure 1: Computational domain.

antee along the interface Γ_I the continuity of the particle velocity and the pressure

$$\mathbf{n}_I \cdot (\mathbf{u}_1{}' - \mathbf{u}_2{}') = 0\,; \quad p_1 - p_2 = 0\,. \quad (5)$$

Now, since we use in Ω_{a2} the scalar acoustic particle velocity Φ, we may rewrite (5) by

$$\mathbf{n}_I \cdot (\mathbf{u}' + \nabla\Phi) = 0\,; \quad p' - \rho_0 \frac{D\Phi}{Dt} = 0\,. \quad (6)$$

According to a Mortar ansatz, we force the continuity of the particle velocities in a strong form by introducing the Lagrange multiplier $\lambda = -\mathbf{n}_I \cdot \mathbf{u}_1{}' = \mathbf{n}_I \cdot \nabla\Phi$ and will fulfill the continuity of the pressures in a weak sense

$$\int_{\Gamma_I} \mu \left(p_1 - \rho_0 \frac{D\Phi}{Dt}\right) ds = 0\,.$$

We in-cooperate these relations into the weak formulations of (1) - (3) and arrive at

$$\begin{aligned} &\frac{1}{\rho_0 c_0^2}\frac{\partial}{\partial t}\int_{\Omega_1} p'\, \varphi\, \mathbf{d}\Omega - \int_{\Omega_1} \mathbf{u}'\, \nabla\varphi\, \mathbf{d}\Omega + \int_{\Gamma_I} \varphi\lambda \mathbf{d}s \\ &+ \frac{1}{\rho_0 c_0^2}\int_{\Omega_1} \overline{\mathbf{u}} \cdot \nabla p'\, \varphi\, \mathbf{d}\Omega = \frac{1}{\rho_0}\int_{\Omega_1} q_c\, \varphi\, \mathbf{d}\Omega \end{aligned}$$

$$\begin{aligned} &\rho_0 \frac{\partial}{\partial t}\int_{\Omega_1} \mathbf{u}' \cdot \boldsymbol{\psi}\, \mathbf{d}\Omega + \rho_0 \int_{\Omega_1} (\overline{\mathbf{u}} \cdot \mathbf{u}')\, \nabla \cdot \boldsymbol{\psi}\, \mathbf{d}\Omega + \int_{\Omega} \nabla p' \cdot \boldsymbol{\psi}\, \mathbf{d}\Omega \\ &- \int_{\Gamma_I} (\overline{\mathbf{u}} \cdot \mathbf{u}')\boldsymbol{\psi} \cdot\, \mathbf{n}_I\, \mathbf{d}s = \rho_0 \int_{\Omega} \mathbf{q}_m \cdot \boldsymbol{\psi}\, \mathbf{d}\Omega \end{aligned}$$

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int\limits_{\Omega_2} q\Phi\,\mathrm{d}\Omega + \int\limits_{\Omega_2} \nabla q\cdot\nabla\Phi\,\mathrm{d}\Omega$$
$$+\frac{2}{c_0^2}\frac{\partial}{\partial t}\int\limits_{\Omega_2} q\overline{\mathbf{u}}\cdot\nabla\,\Phi\,\mathrm{d}\Omega - \frac{1}{c_0^2}\int\limits_{\Omega_2}(\overline{\mathbf{u}}\cdot\nabla q + q\nabla\cdot\overline{\mathbf{u}})\,(\overline{\mathbf{u}}\cdot\nabla\Phi)\,\mathrm{d}\Omega$$
$$-\frac{1}{c_0^2}\int\limits_{\Gamma_\mathrm{I}} q\,(\mathbf{A}\nabla\Phi)\cdot\mathbf{n}_\mathrm{I}\,\mathrm{d}s + \int\limits_{\Gamma_\mathrm{I}} q\lambda\,\mathrm{d}s = 0$$
$$\int\limits_{\Gamma_\mathrm{I}} \mu\left(p' - \rho_0\frac{D\Phi}{Dt}\right)ds = 0\,.$$

with $\mathbf{A} = (a_{ij}) = (\overline{u}_i\overline{u}_j)$ and appropriate test functions φ, $\boldsymbol{\psi}$, q and μ. For the physical quantities p', Φ and test functions φ, q, μ we can use standard continuous Lagrange finite elements, whereas for $\mathbf{u}'$ and $\boldsymbol{\psi}$ we need discontinuous finite elements. With this mixed ansatz we arrive at a stable FE formulation [4].

Numerical Tests

To test our multi-model formulation we consider a setup as displayed in Fig. 2. Over the whole computational domain we assume a shear flow with Mach 0.2 and define a dipole source at the center of the domain. Figure 3 shows the computed acoustic pressure at two characteristic time steps. As can be seen, there are no reflections at the interface between the inner domain computed by solving APE and the outer region, where we solve for Pierce equation. Further investigations showed that even for higher Mach numbers the Mortar formulation works.

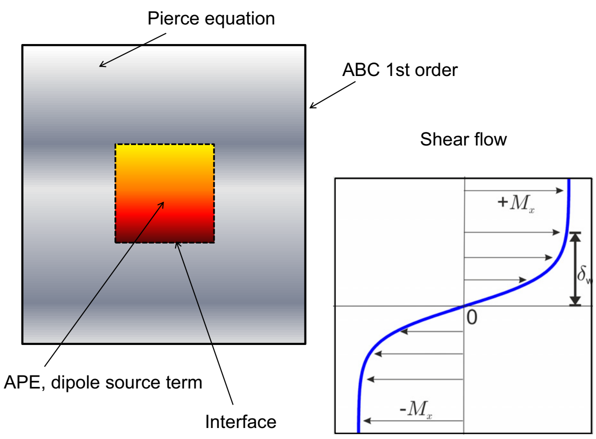
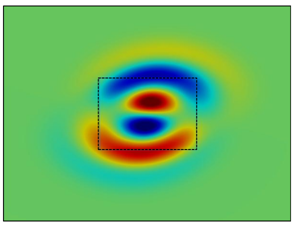
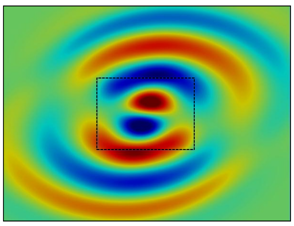


Figure 2: Computational domain.



(a) Acoustic pressure at t_1



(b) Acoustic pressure at t_2

Figure 3: Computed acoustic pressure for two characteristic time steps.

Since for most applications the inner region, where we solve for APE (in 3D four unknowns), can be kept small, we are able to strongly reduce the computational complexity, and therefore arrive at strongly reduced CPU times.

Conclusion and Outlook

We have presented a new multi-model approach for computational aeroacoustics. Therewith, we model the flow induced sound by the acoustic perturbation equations within the turbulent flow region and use the Pierce equation for the outer region. By applying a Mortar ansatz and appropriate FE discretizations for the equations, we could successfully demonstrate the correctness of the new scheme.

Currently, we investigate in the mathematical analysis of the whole scheme and apply it to practical relevant applications, e.g., the computation of the sound generated by the flow around car side mirrors.

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