Choosing Optimal Delays for Feedback Delay Networks

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Introduction

Feedback delay networks (FDNs) are often used in the context of artificial reverberation and are a class of sparse IIR filters. FDNs are based on a feedback loop with multiple channels containing delay elements, as well as a mixing matrix providing a connection between the channels. An example of a simple FDN is shown in Figure 1. For a practical FDN implementation, many parameters have to be chosen: the number of channels, the mixing matrix, the delays, several gains, and, in the case of FDNs that implement a frequency-dependent reverberation time, filters for each channel. Some of these parameters can be computed from room impulse response properties [2], while for others popular choices exist, e.g. using a Hadamard matrix as the mixing matrix.

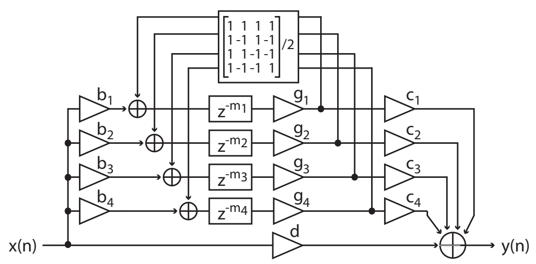

Figure 1: Block diagram of a simple feedback delay network with four channels. Each of the channels contains a delay line, and the channels are arranged in a feedback loop in which a mixing matrix provides connections between all channels.

However, there are only few rules for the choice of delays, even though the delays affect the reverberator's coloration [1], as well as its mode and echo densities [2]. Smith computed [4] a lower limit for the sum of all delays in order to assure a minimum mode density: $\sum_{i=1}^{N} m_i \geq 0.15 \cdot \mathrm{RT60} \cdot f_s$, where N is the number of channels, m_i are the delays measured in samples, RT60 is the reverberation time in seconds and f_s is the sampling frequency in Hertz.

A commonly applied rule for choosing delays is to select them to be mutually prime. This rule was first presented in Schroeder's seminal publication on artificial reverberation [3] with the argument that it reduces echo superposition. Despite structural differences between the Schroeder reverberator and FDNs, notably the absence of a mixing matrix in the former, this argument has been taken up by many working in the field of FDNs [2, 4].

This paper shows that mutually prime delays only marginally reduce echo superposition in FDNs with non-sparse mixing matrices, and shows a way of selecting delays using a measure for potential echo superposition and an optimization method based on this measure.

Delays and echo superposition in FDNs

The mutually prime delays criterion can be justified easily for an FDN without a mixing matrix, which has the same loop topology as the Schroeder reverberator. Due to the independent feedback loops, the impulse response will contain nonzero samples only at $n_I = km_i$, where $k \geq 1$ is an integer, and m_i is the delay of channel i. The first sample in the impulse response where more than one feedback loop produces a nonzero output must therefore be the least common multiple of two delays m_i and m_j. Given that the delays are mutually prime, this corresponds to $m_i m_j$. Therefore, the product of the two smallest delays determines the first time instant when echo superposition occurs. The effect of echo superposition is shown in Figure 2: echos where superposition occurs exceed the exponentially decaying envelope defined by the other echoes by a factor of two. This may be perceived as an increase in roughness of the sound.

For FDNs with a non-sparse mixing matrix, nonzero samples occur at $n_M = \sum_{i=1}^{N} a_i m_i$, where $a_i \geq 0$ are integers. The set of all possible values of n_M is therefore a superset of all possible values of n_I. This means that many more possibilities exist for echo superpositions, which will also occur for example when $a_1 m_1 + a_2 m_2 = a_3 m_3$. This is often the case even with mutually prime m_i and small values for a_i, e.g. for $m_1 = 127$, $m_2 = 251$, $m_3 = 629$, and $a_1 = 1$, $a_2 = 2$, $a_3 = 1$. Echo superpositions will inevitably happen in FDNs and using mutually prime delays avoids only a negligible subset of echo superpositions. This is illustrated in Figure 3 where no visible reduction of echo superpositions occurs due to the use of mutually prime delays.

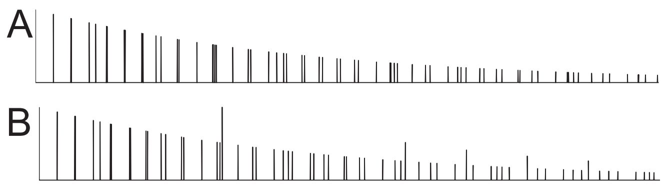

Figure 2: Impulse responses for an FDN without mixing matrix. **A**: using mutually prime delays. **B**: using non mutually prime delays, resulting in echo superposition.

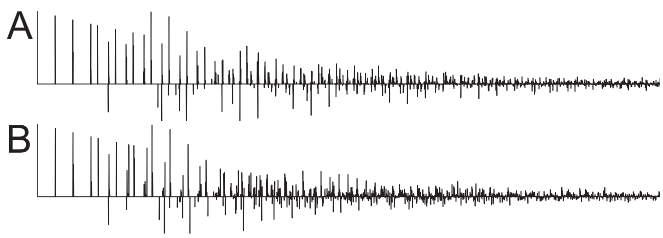

Figure 3: Impulse responses for an FDN with mixing matrix. **A**: mutually prime delays. **B**: non mutually prime delays.

Quality metric based delay optimization

Given that using mutually prime delays only has a marginal effect on echo superposition in FDNs with mixing matrices, a new approach to choosing FDN delays is proposed, based on the optimization of a quality metric derived directly from the delays.

Potential nonzero samples

The approach presented here is to consider, based on the delays, the potential nonzero samples in the impulse response $h(n)$. Since $h(n)$ can be nonzero only if there exists a combination of integers a_i such that $n = \sum_{i=1}^{N} a_i m_i$, quality measures for the delays m_i can be implemented using the number of a_i combinations for each n, denoted $C(n)$. Nonzero values of $C(n)$ do not necessarily imply that $h(n) \neq 0$ as, depending on the mixing matrix, two different paths through the FDN may result in signal components that cancel each other out.

An algorithm to compute $C(n)$ for $n \leq M$ is described in the following MATLAB code (however, for efficiency reasons, the optimization was performed using a C implementation).

```
am=floor(M./m)+1; C(1:M)=0; a(1)=1; a(2:N)=0;
while sum(a)>0
  d=sum(a.*m);
  if d<=M
    C(d)=C(d)+1; ainc=1;
  else
    ainc=find(a>0,1)+1; a(ainc-1)=0;
  end
  if ainc<=N, a(ainc)=a(ainc)+1; end
  for i=2:N
    a(i)=a(i)+floor(a(i-1)/am(i-1));
    a(i-1)=mod(a(i-1),am(i-1));
  end
  a(N)=mod(a(N),am(N));
end
```

Quality measures and optimization

Based on $C(n)$, a quality measure q was defined with the goal to improve the echo density in the beginning of the impulse response: $q = \sum_{n=1}^{M} w(C(n))$, where $w(c)$ is a weighting function modeling the probability that c delay combinations cancel each other out. Note that $w(0) = 1$.

An iterative optimization algorithm was used to find the combination of delays m_i that minimizes q. The algorithm starts out with an initial set of delays. At each step of the algorithm, each m_i is varied within an interval defined by a target interval for $\sum_{i=1}^{N} m_i$ and the quality measure is computed. The combination of delays with the best quality measure is used in the next iteration step. The algorithm stops when no more improvement can be achieved.

Optimization results

For the quality measure q, a typical optimization run changes the delays as shown in Figure 4. It was observed that small delays tend to become smaller and large delays tend to become larger. While the optimization of the quality measure was successful in the sense that the number of nonzero samples in the beginning of the impulse response was significantly increased, the split into very short and much longer delays also leads to wildly varying amplitudes of the nonzero samples in the impulse response. A positive perceptual effect of the optimization could not be proven so far.

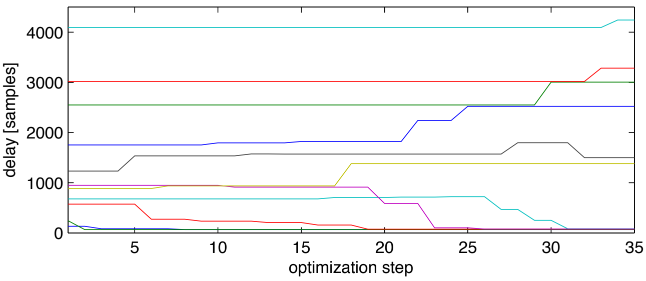

Figure 4: Delays m_i during an optimization run.

Conclusion

It was shown that for FDNs with non-sparse mixing matrices, the common practice of using mutually prime delays does little to avoid echo superposition or cancellation. Therefore, this paper proposes to drop the mutually prime criterion and to apply an optimization method to find suitable delays. A quality measure was derived from the delays and was optimized, starting from an initial set of delays, in order to improve the echo density in the beginning of the impulse response. A significant increase in echo density was observed due to the delay optimization. However, the perceptual difference did not reflect this increase, which can be explained by the fact that nonzero samples in the impulse response had greater amplitude variations when using the optimized delays, rather than the original delays. While the use of non mutually prime delays in FDNs is very promising, more research is needed on the delay optimization.

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