A Method for the Generation of Subharmonic Frequencies

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Introduction

There exists a number of methods to generate sub-harmonic frequencies for a given audio signal [1-4]. Pitch-changing algorithms which work in the frequency domain [1] require FFT/IFFT operations which result in considerable algorithmic delay (latency). The desired frequency components arise only after a certain reaction time which makes the methods less usable for real-time applications of musicians. Other disadvantages of some of the known methods are the emergence of significant distortion products.

The main scope of the method described in this paper is the real-time application for musicians or singers.

Algorithm

The method which will be described in the following avoids the disadvantages of former approaches and can be used in analog circuitry or as an algorithm in a DSP.

The method that we propose builds up on a basic formula which can be found in math text books [5]:

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}} \tag{1}$$

where $x = 2\pi f t$.

So for a given cosine-shaped signal a signal whose frequency is right down by one octave is generated when this formula is applied. It is stated that the original signal must be a pure tone for the formula to work in an exact way. When a signal with a broader spectrum is to be processed, one (typically the fundamental) frequency must be extracted before applying the algorithm.

In figure 1, the signal processing of the single processing steps is illustrated. Figure 1 a) shows a single-frequency signal which may have been derived by filtering the input signal. Figure 1 b) shows the argument of the square root in equation (1), where amplitude is halved and a constant component has been added. Now, in figure 1 c) the square root function has been applied. The effect can be observed that small values are emphasized, and the higher values are a kind of "compressed" so that from one (raised) period of the original signal a half-wave is formed. It can however be observed that all the half-waves reach in the positive direction. Therefore, in a last step every second half-wave is sign-inverted (the (+/-)-operator) and as a result we see a cosine signal with half the frequency of the original signal (Figure 1d)).

Challenges that have to be overcome to apply this formula are

- Extracting a fundamental, single frequency from a mixture of frequencies, i.e. a general, wideband signal. This will be achieved by appropriate pre-processing such as lowpass or bandpass filtering.
- Capturing the rectified half-waves and "counting" them to control a polarity-inverting processing stage. Note that the half-waves are an analog signal while counting is a term from digital processing. In practice, the partially processed signal from figure 1 c) will maybe not look so clean or mathematically perfect as in the figure. It will probably need some means to make polarity switching a robust process.

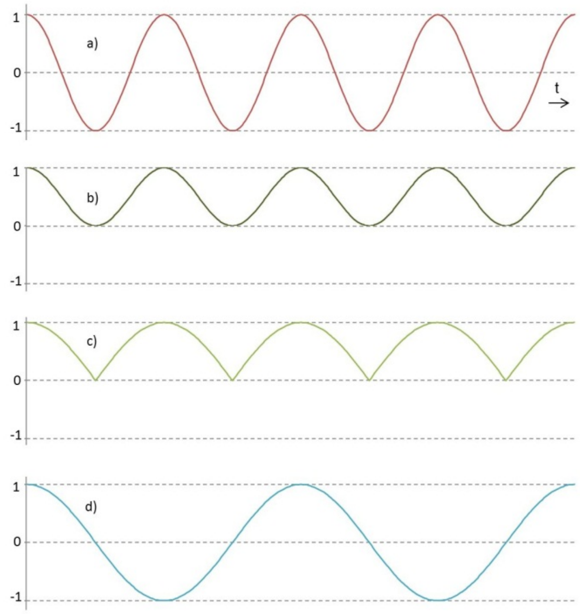

Figure 1: Illustration of signal processing (see text)

Implementation

The development of a signal processing chain implementing equation (1) is quite straightforward. The overall processing consists of the following steps:

- Pre-filtering of the input signal to extract a single frequency, typically the lowest contained frequency
- Formation of the envelope of the signal. An input signal is rectified by a full-wave rectifier, and this signal is smoothed to obtain the envelope of the signal, corresponding to the "1" in the formula.
- A summing stage adds the input signal to the envelope and the result corresponds to the term "$(1 + \cos(2\pi ft))$"
- A square root function is applied whose argument is always positive in this case. In the analog domain,

the root extractor can be approximated, for example, as a circuit in the form of a non-linear diode/resistor network. In digital systems, programming the square root function is almost trivial.

- A stage that we call "variable sign amplifier" which, in dependence of a control signal, transfers the input signal either in phase (0° phase shift) or phase inverted (180° phase shift) to its output. As a result, this amplifier implements the polarity switching (+/-) function which is part of the above formula. In a prototype, this variable amplifier was realized as an op-amp differential amplifier with two light-dependent resistors (LDR) in its peripheral circuitry.
- In a last step, the control signal which switches the sign of the variable amplifier has to be won. For that purpose, a trigger circuit, preceded by an optional signal conditioning stage, triggers a binary counter which divides the input frequency by an integer factor, namely 2 for a one-octave sub-harmonic. There can be different forms of the trigger circuit depending on the type of the processed signal. In some cases, it has shown to be advantageous to use a zero-crossing detector with a small hysteresis.
- There can be an optional post-filtering of the newly generated sub-harmonic signal to eliminate possible artifacts or undesirable "higher" frequency components in this signal.
- In a device for use in the studio or on stage, the original signal will be mixed with the sub-harmonic in a "dry/wet" manner, i.e. adding more or less sub to a given signal.

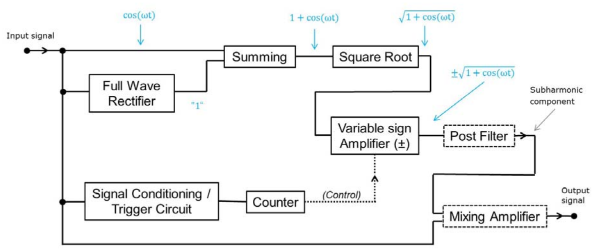

Figure 2: Block Diagram of the Sub-Harmonic Frequency Generation Method

Results

To prove the described method, both an analog hardware implementation and a software version (a VST plugin) have been built. The main difference between these devices lies in the pre-processing (signal conditioning) stage. The hardware solution uses separate tunable highpass and lowpass filters (24dB/oct.) to extract the fundamental frequency. Opposed to that, the available software plugin has a bandpass filter where center frequency and quality factor can be varied.

Both version work well depending on the audio material. Figure 3 gives one example where the software version was used. Here, a post filter in the form of a 100 Hz lowpass was used which in this case even would not have been required.

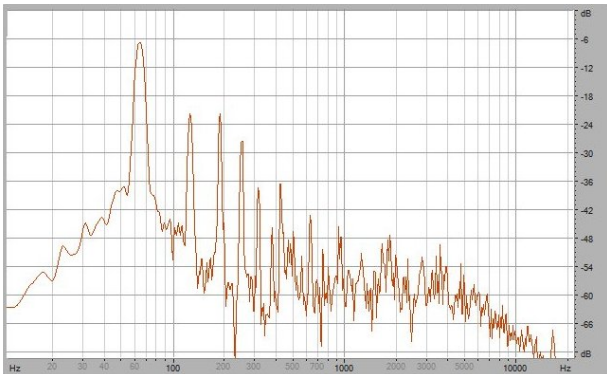

Figure 3a): Spectrum of a Synthesizer bass note (fundamental frequency = 65 Hertz).

Figure 3b): Spectrum of generated sub-harmonic signal (calculated by a 16384-point FFT and a Blackman-Harris window function)

It can be concluded that the functionality of the proposed algorithm has been proven by the first prototypes and the devices at hand.

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